LASER BEAM ATTENUATION IN THE LOWER ATMOSPHERE

BY

R. M. LANGER

The J. R. M. Bege Company
4 Court Street Place
Arlington, Massachusetts

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Abstract

Small angle spreading, aerosol scattering and molecular absorption are considered the important mechanisms for the weakening of a laser beam in the open atmosphere. Three different transmission laws are worked out for these three mechanisms. Both the physical principles and the numerical values encountered in the lower atmosphere are discussed and illustrated. Random density fluctuations in the turbulent atmosphere are discussed as the cause of small angular deflections in a narrow pencil of light. Beam attenuation due to atmospheric aerosol scattering is treated for an aerosol size distribution described by the sum of two inverse powers of the droplet radius. Laser beams can help find the parameters of such distributions. Molecular absorption is examined in terms of the narrow infrared lines of water vapour. An effort is made to present this difficult topic in as simple and useful a form as is compatible with the observational material. The formulae are designed to make it possible to estimate in detail how the atmosphere would weaken a laser beam under a wide variety of conditions. It is found that some effects are serious even at short ranges of a few meters, while in favourable circumstances, laser signals would not be drastically attenuated out to any practical distance in the lower atmosphere. ( )

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1. Atmospheric Optics

1.1 Basic Quantities

The utility of lasers for communication purposes or as probes for atmospheric investigations will depend considerably on the small geometrical cross section of a laser beam, its narrow spectral range, and its high intensity. These are regarded as outstanding laser beam qualities which guide the present discussion of atmospheric optics. The propagation of coherence will not be discussed here. The atmosphere will be treated as a clear medium in which there are suspended randomly scattered discrete centers of disturbance which may be classified according to size. The scale or size parameter x will involve the wave length of the incident light λ and the dimensions of the region in which a refractive index deviation occurs. The definition of x is

$$x = \frac{2\pi a}{\lambda} \tag{1.1.1}$$

For a spherical or cylindrical particle, a would be the radius; for some other particle, irregular in shape or in refractive index distribution, it would perhaps be a mean effective radius which could readily be defined more specifically for the case at hand. The particles or regions in the atmosphere

ch interact with the light only by way of changes in refractive index m along the path. Normally, the refractive index of the atmosphere surrounding the disturbing element may be taken as unity and the disturbances produced ... be proportional by the quantity (m - 1) which measures the change in refractive index of the particle with respect to the atmosphere. (The quantity m will be complex when it is necessary to describe energy exchanges between the light and the disturbing particles.) It should be noted that when precise values of wave lengths λ are involved, the vacuum wave length λ_{vac} is connected to the wave length λ in air by the formula

$$\lambda_{vac} = \lambda \, m_{atmos.} \qquad (1.\ldots)$$

where $m_{atmos.}$ is the actual refractive index of the atmosphere which at ... is not exactly unity but about 1.000293. It varies roughly with local atmospheric density δ according to the equation

$$(m_{atmos.} - 1) = 2.93 \times 10^{-4} \frac{\delta}{\delta_1} \qquad (1.1.3)$$

where δ_1 is the NTP density of the atmosphere.

The interesting ranges of x values for our purposes lie in three regions which will be treated separately.

A. Particles or regions large compared with wave length; $x \gg 1$

B. Particles comparable to λ; $x \approx 1$

C. Particles small compared to λ; $x \ll 1$

1.2 [illegible] 1

[illegible] gory A [illegible] material falling [illegible] rain, hail, [illegible], soot or poll[illegible] In [illegible] dispersions pro[illegible] atomic detonations. [illegible] cat[illegible]ry, reference is made to fluctuations in local [illegible] associated with turbulence where temperature, humidity, or pressure gradients occur.

1.3 [illegible] Length Comparable with Particle Size; x [illegible] 1

[illegible] aerosols constitut[illegible] range in radius [illegible] from [illegible] 0.1 [illegible] cron to many tens of [illegible]. Their settling rate under gravity negligible, but they move vertically because of convection curr[illegible] [illegible] of the suspended mat[illegible] is water, but solids in the form [illegible], dust, biological, vo[illegible] or industrial colloids or salt [illegible]tion [illegible] in atmospheric aerosols. The loss of su[illegible] material [illegible] evaporation [illegible] precipitation [illegible] to limit the liquid water content [illegible] air [illegible] about the amount of water that could be contained in [illegible] form. The source of the liquid gener[illegible]y is the [illegible]. When t[illegible] [illegible] pressure is reached, the aerosol droplets tend to gro[illegible] [illegible] the[illegible].

1. [illegible]ative[illegible] Wave Length; x [illegible] 1

[illegible] small particles in Category C are the most numerous constitu[illegible] of the atmosphere but not i[illegible] respects the most effective. They are predominantly the individual [illegible] of the major permanent gases nitrogen and oxygen. Their scattering properties were described earliest by Rayleigh, but rarely is the Rayleigh scattering comparable with that due to the less numerous aerosols [illegible] Category B. It accounts for the blue color of the sky, but the com[illegible] observed scattering called ha[illegible] and

limited [illegible] op[illegible] and other minor and [illegible]ry elements [illegible] in the lower [illegible]ph[illegible]. The molecular absorption due to oxygen and nitrogen is in the ultraviolet part of the spectrum and will not concern us here because laser beams so far have been restricted to the visible and infrared portions of the spectrum. The visible and infrared absorption lines observed in the atmosphere are due predominantly to the minor constituents, water vapor and carbon dioxide molecules. The present paper is devoted primarily to this aspect of atmospheric studies partly because of its importance and intrinsic interest for laser beams, and partly in order to present a treatment of the subject which is more convenient and more instructive than the methods currently used in discussing molecule line absorption in water, carbon dioxide, and other polyatomic gases.

2. Transmission Laws

2.1 Fluctuations in Refractive Index

Laser beam effects in a patchy atmosphere are examined here. These are Category A phenomena in the sense that we consider the atmosphere to be subdivided into cells whose refractive index varies from its mean value over regions which may be meters across. Thus, $x = 2\pi a/\lambda \gg 1$. Other category A phenomena involving more palpable objects such as raindrops, snowflakes, fallout particles will be treated elsewhere as tools for atmospheric probing, but their behavior relative to atmospheric transmission is adequately covered as a limiting case of Category B. The typical source of the patchiness treated here is the turbulence due to overturning moments under certain conditions in the atmosphere. Cells of circulating air form spontaneously. These cells are regarded as the source of scintillations and bad seeing in astronomical and terrestrial observations. They occur high and low in the atmosphere over land and ocean. Other micrometeorological effects also cause patchiness in refractive index. Thermal gradients with or without wind, pockets of high humidity, shade from insolation or roughness of terrain may cause variations in density. Oscillations up to hundreds of cycles per second as well as translation of these cells are known to occur. They vary in strength and frequency but are always present to some degree to the extent they limit the useful aperture for optical astronomical telescopes to 300 inches or less even at the most favorable mountain sites. In a somewhat different way, they may limit the utility of laser beam communications at sea level or low altitudes.

The light flux in astronomy or in meteorological work usually covers a broad region. The wave front may be regarded as flat and infinite in extent.

normal to the direction of propagation. Sidewise displacement of the beam means nothing and only direction or phase counts. A laser beam is narrow both in angle and in cross section. It is more like a pencil or ray in an optical instrument. Sidewise as well as angular displacement can take a photon out of the useful region of observation. The question asked here is--how much does a patchy atmosphere reduce the light energy received from a distant laser? The answer takes into account the many small deflections caused by atmospheric refraction. They cause the apparent image size of the laser to be enlarged by angular deflection and they also spread the light over an area greater than the original laser beam would cover. If the path length is great, the energy received within a given aperture may be noticeably reduced by beam spread. We consider next the situation where numerous scatterings take place, but since each deflection is very small, the beam remains within a narrow cone even when it is large compared with the receiving instrument. It is evident that geometrical optics is adequate for our purposes.

2.2 Ray Curvature in a Refractive Index Gradient

A laser beam traversing a region where there is a refractive index gradient ∇m will be deflected along its path according to the formula

$$\frac{1}{\rho} = \nabla_n m \qquad (2.2.1)$$

where ρ is the radius of curvature of the path and $\nabla_n m$ is the component of the gradient of the regractive index m, normal to the ray path. In a small length of path δs, a small normal gradient $\nabla_n m$ would deviate a ray through an angle

$$\delta\theta_1 = \delta s \cdot \nabla_n m \tag{2.2.2}$$

and would produce a sidewise displacement

$$\delta y_1 = \frac{1}{2} \nabla_n m \cdot (\delta s)^2 \tag{2.2.3}$$

The first thing to note about these quantities is that they are small and that our assumption of narrow pencils or small cross section is justified. The next thing to show is that the deviations are not always negligible and thus are worthy of further examination.

Suppose that we have a horizontal ray moving across a vertical temperature gradient of say $1^\circ C$ per meter height. Then there would be a density gradient and consequently by equation (1.1.3), a transverse refractive index gradient

$$\nabla_n m \approx 2.9 \times 10^{-4} \times \frac{1^\circ}{290^\circ} \times \frac{1}{100\text{cm}} \approx 10^{-8}\ \text{cm}^{-1} \tag{2.2.4}$$

The angular deflection in, say, 10 meters given by equation (2.2.2) would be

$$\delta\theta_1 \approx 10^3 \times 10^{-8} = 10^{-5}\ \text{radians} \approx 2\ \text{seconds of arc} \tag{2.2.5}$$

and by equation (2.2.3) a deflection

$$\delta y_1 \approx 5 \times 10^{-3} \text{ cm} \tag{2.2.6}$$

A laser beam might be of the order of 10^{-4} to 10^{-3} radians in angle and about 1 cm in cross section diameter, and so, these deflections are seen to be small. The atmosphere could well have cells 10 meters across with the density gradients mentioned, and so in a kilometer path, the resultant total deviations would perhaps not be trivial. If the gradients were randomly arranged, then in a path of N cells, the resultant deviations would be $\sqrt{N}$ times as great as the unit values given in (2.2.5) and (2.2.6). For a 1 km. path and 10 meter cells

$$N \approx 100$$

and we would have

$$\Delta\theta_{100} \approx \sqrt{N}\,\delta\theta \approx 10^{-4} \text{ radians}$$

and

$$\Delta y_{100} \approx \sqrt{N}\,\delta y_1 \approx 0.05 \text{ cm.}$$

In a laser with, say, a 5×10^{-4} radian beam, the illumination at 1 km. would cover a 50 cm. spot and a 0.05 cm. enlargement would not count. However, the spot enlargement would be determined by the total angular

deflection which, in this example, is 1/5 of the original beam angle. Thus, a 20 to 40 percent reduction in energy flux through a given aperture would be expected. This would be a time average. Brief fluctuations of the nature of scintillations would be much greater.

2.3 Transmission Law For a Patchy Atmosphere

The preceding paragraph is a basis for writing down the transmission law for an atmosphere whose only optically active element is a random assembly of cells each containing a refractive index gradient of mean effective value ∇m. Take a mean cell size S and ask how the energy falling on a high resolution light collecting instrument would fall off with range R along the path of a laser beam whose divergence in radians is α. The distance R is supposed great enough so that the geometrical beam at R would be large in cross section compared with the aperture of the light sensor. As discussed in the preceding paragraph, the effect of the patchiness is to enlarge the beam diameter at range R from

$$R\alpha \tag{2.3.1}$$

to approximately

$$R\alpha + R\,\Delta\theta \tag{2.3.2}$$

where $\Delta\theta$ is the cumulative deflection of all the cells along the path R. Along the path R, there are about R/S cells, each causing a small angular deflection $\delta\theta$ according to (2.2.2). The cumulative deflection $\Delta\theta$ due to R/S cells is then

$$\Delta\vartheta \simeq \delta\sqrt{\overline{m}}\,\sqrt{\frac{R}{\delta}} = \sqrt{\overline{m}}\sqrt{R\delta} \qquad (2.3.3)$$

in notation slightly different from that used before. The enlarged laser beam diameter (2.3.1) at range R is therefore

$$R\left(\alpha + \sqrt{\overline{m}}\sqrt{R\delta}\right) \qquad (2.3.4)$$

and the energy flux per unit area received there is proportional to the inverse square of this. It is now obvious that the factor T(R) by which the energy is reduced by transmission through the patchy medium is

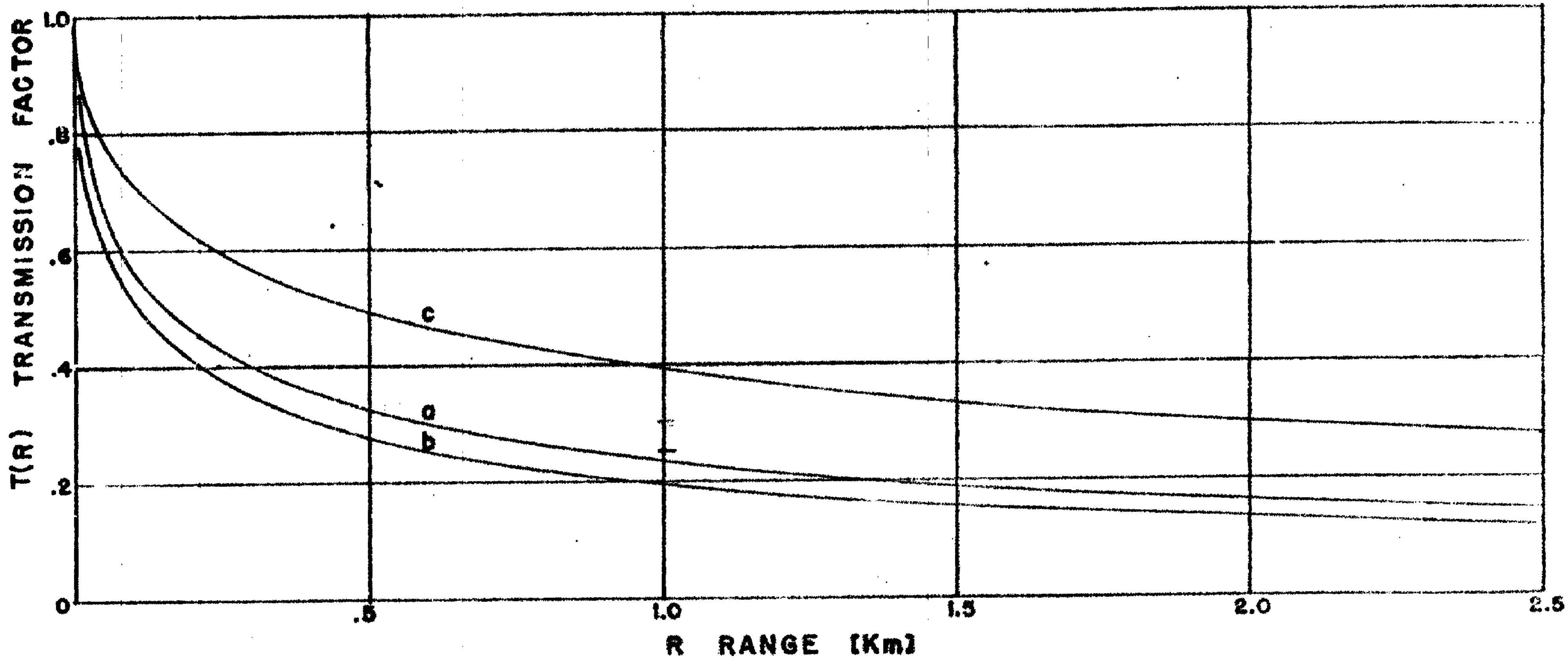
$$T(R) = \frac{(R\alpha)^2}{R^2\left(\alpha + \sqrt{\overline{m}}\sqrt{R\delta}\right)^2} \qquad (2.3.5)$$

The final expression of the transmission law (I) for a patchy medium with cell size S is thus

$$\text{(I)} \qquad T(R) = \frac{1}{\left(1 + \frac{\sqrt{\overline{m}}}{\alpha}\sqrt{RS}\right)^2} \qquad (2.3.6)$$

I. $$T(R) = \frac{1}{\left(1+\frac{\nabla m}{\alpha}\sqrt{RD}\right)^2} \qquad (2.3.6)$$

CURVE	∇m	α	S
a	10^{-8} cm.$^{-1}$	3×10^{-4} radians	10^3 cm.
b	2×10^{-8}	3×10^{-4} radians	10^3
c	10^{-9}	3×10^{-4} radians	10^4



TRANSMISSION LAW FOR PATCHY MEDIUM

Figure (2.3.1)
p. 15

Here $\overline{\nabla m}$ = the mean effective gradient of the refractive index m in a cell. (cm^{-1})

α = Angular width of laser beam in radians.

S = Mean cell size (also spacing) in cm.

R = Range of laser beam to observing instrument. (cm.)

The derivation of this formula has disregarded various factors of the order of unity. They may be considered hidden in the definition of $\overline{\nabla m}$, the gradient of refractive index, or in S, the size of the cell. Since these both occur as factors of $\sqrt{R}$, it is very easy to compute T(R) for various assumed values of cell size S and gradient $\overline{\nabla m}$ or laser beam angle α. The computation amounts merely to a change of scale of R and can be done by mental arithmetic. Figure (2.3.1) shows three curves of T(R) for values of the parameters shown in Table (2.3.1). The transmission for sufficiently

Table (2.3.1)

Parameters For The Curves In Figure (2.3.1)

Curve	α (radians)	$\overline{\nabla m}$ (cm^{-1})	S(cm)
a	3×10^{-4}	10^{-8}	1,000
b	3×10^{-4}	2×10^{-8}	1,000
c	3×10^{-4}	10^{-9}	10,000

large ranges R varies inversely as RS. Thus the energy fall off with distance along a laser beam in a patchy medium goes inversely with R^3. In the strato-sphere where long range, almost horizontal, communications might be of interest

the cell size S might be much larger. Thus even with much smaller refractive index gradients, the attenuation of a laser beam could be significant. The impression gained from Figure (2.3.1) is that turbulence and other patchiness is always likely to degrade laser beams but is not, by itself, a primary limitation in long distance communications.

2.4 Scattering By Water Droplets

The most abundant aerosol in the atmosphere is liquid water in the form of small droplets. These droplets in cloud, haze, spray or mist will be used as examples of Category B where the particle dimensions are comparable with wave length ($x = 2\pi a/\lambda$). The very extensive literature on this subject has been summarized together with many of his own results by H. C. van de Hulst[2.4.1]. The exposition he presents extends to very small and very large values of x so all categories A, B and C are included to some extent, but there are many aspects of the general problem that are still in an unsatisfactory state. The theory for refractive index m which theoretically may be of any magnitude real or complex is readily amenable to computation and analysis only when

$$|m - 1| << 1 \qquad (2.4.1)$$

The present article will be restricted almost entirely to this condition. Fortunately, the formulae developed under the restriction (2.4.1) hold well enough for our purposes even when

$$(m - 1) = 0.33 \qquad (2.4.2)$$

(2.4.1) Light Scattering by Small Particles by H. C. van de Hulst; - John Wiley & Sons; New York (1957).

as is the case for visible light incident on liquid water droplets. For laser beam communication studies, it will surely be necessary at sometime to go beyond (2.4.1) because many laser systems are in the infrared where liquid water shows anomalous dispersion with large, rapidly varying, complex values of $(m - 1)$. These will be postponed. Water droplets in cloud, fog, and mist are most efficient as scatterers for near visible light and their absorption properties are usually overshadowed by this scattering. For narrow spectral regions in the infrared, it may be necessary to take absorption into account and this will be done in another report where particular lasers come up for discussion. For long range laser beams in the stratosphere where little water is present, it will again be necessary to consider absorbing aerosols. Here, transmission through scattering atmospheres without absorption will be contemplated. Molecular absorption in laser beams will be treated later in this article under Category C phenomena.

A light beam with a flux density I in watts per square centimeter would deliver $\pi a^2 I$ watts to the projected area of a drop of radius a. The light scattered by the drop could be written

$$\pi a^2 I Q_s \qquad (2.4.3)$$

where Q_s would be a measure of the effectiveness of the drop as a scatterer. The cross section for scattering C_{scat} for the drop is defined as

$$C_{scat} = \pi a^2 Q_s \qquad (2.4.4)$$

and since zero absorption is assumed, this is the total energy lost from the incident beam. A random collection of such drops separated by distances large compared with a, will have a combined attenuation (or extinction) cross section $C_{ext} = C_{scat}$ given by

$$C_{ext} = \pi a^2 p Q_s \tag{2.4.5}$$

where p is the number of drops. If there are N drops per cubic centimeter in the atmosphere, the transmitted fraction after traversing a path of length R would be

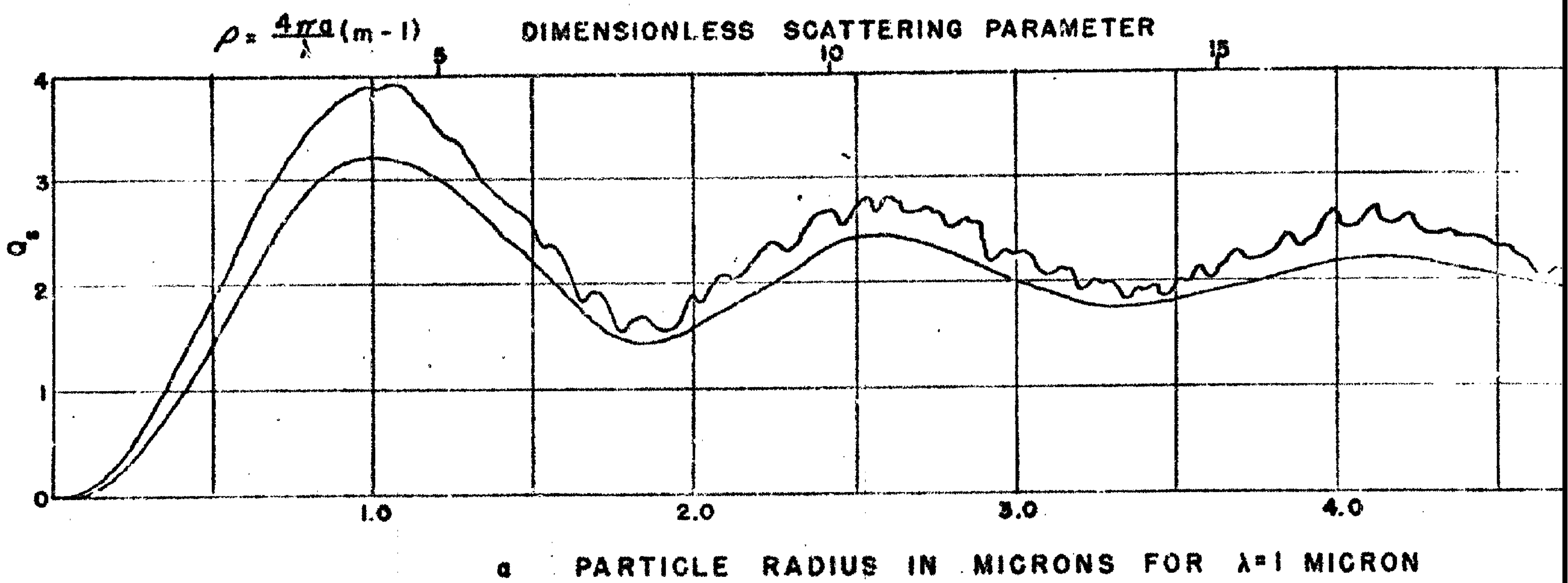
$$T(R) = e^{-\pi a^2 N Q_s R} \tag{2.4.6}$$

This is not yet suitable as a transmission law because real aerosols are not composed of particles all of the same size. Moreover, Q_s is a function both of a and of λ and it must be discussed in some detail in order to get a useful formula. The most suitable, explicit expression for Q_s is to be obtained from van de Hulst (1957), page 167 or 176. Formulae for his quantities $A(\rho, o)$ or Q_{ext} can be applied for our purposes in the form

$$Q_s = Q_{ext} = 2 - \frac{4}{\rho} \sin\rho + \frac{4}{\rho^2}(1 - \cos\rho) \tag{2.4.7}$$

where

$$\rho \equiv 2x(m - 1) \equiv \frac{4\pi a}{\lambda}(m - 1) \tag{2.4.8}$$



Q_3 SCATTERING EFFECTIVENESS PER UNIT AREA $[cm^{-2}]$

Figure (2.4.1)
p. 19a

A plot of Q_s in (2.4.7) is shown in Figure (2.4.1) as a function of particle radius a for $(m - 1) = 0.33$ and $\lambda = 1$ micron $= 10^{-4}$ cm. The limiting value of Q_s for large ρ is

$$Q_s = 2 \qquad \text{limit } \rho \to \infty \qquad (2.4.9)$$

while for small ρ we can expand the trigonometric terms in (2.4.7) and find

$$Q_s \doteq \frac{\rho^2}{2} \doteq \frac{8\pi^2 a^2}{\lambda^2}(m - 1)^2 \quad \text{limit } \rho \to 0 \qquad (2.4.10)$$

The derivation of Q_s depended on assuming that $x \gg 1$ and so geometrical optics could be used. It is, therefore, not safe to use (2.4.10) for values of a/λ smaller than unity. For small values of $x = 2\pi a/\lambda$, the Rayleigh fourth power law holds. Then

$$Q_s = \frac{16x^2}{27}\frac{\rho^2}{2} \qquad (2.4.11)$$

instead of (2.4.10). This means that (2.4.10) overestimates Q_s for $x \ll 1$.

It is interesting to use these results to examine familiar conditions in the atmosphere. Figure (2.4.1) is somewhat misleading in that it represents the comparative effectiveness of the same number of droplets of different radius. More appropriate would be the comparative effectiveness of a given mass of water droplets as a function of the particle size in which they are dispersed. Thus, we eliminate N in (2.4.6) by means of an

expression for the total mass M of the liquid water drops suspended in unit volume of the atmosphere.

$$M = \frac{4}{3} \pi a^3 \cdot Nd \qquad (2.4.12)$$

where N is a before the number of drops of radius a per unit volume and d is the density of liquid water. The limiting mass of water suspended in droplets is surely closely related to the vapour pressure of water at temperatures the air mass is likely to have experienced in its recent past. For definiteness, we can imagine that the mass M in (2.4.12) is about half the liquid water content of a cubic centimeter of completely saturated water vapour at, say, 73°F. This amounts to assuming

$$M = 10^{-5} \text{ grams/cm}^3 \qquad (2.4.13)$$

The atmosphere could contain much less than this amount under dry conditions, but it could not support much more without precipitation before long. The transmission factor $T(R)$ in (2.4.6) now becomes

$$T(R) = e^{-\frac{3M}{4d} \cdot \frac{Q_s}{a} R} = e^{-0.75 \times 10^{-5} \frac{Q_s}{a} R} \qquad (2.4.14)$$

when (2.4.13) is used for M. Evidently the distance

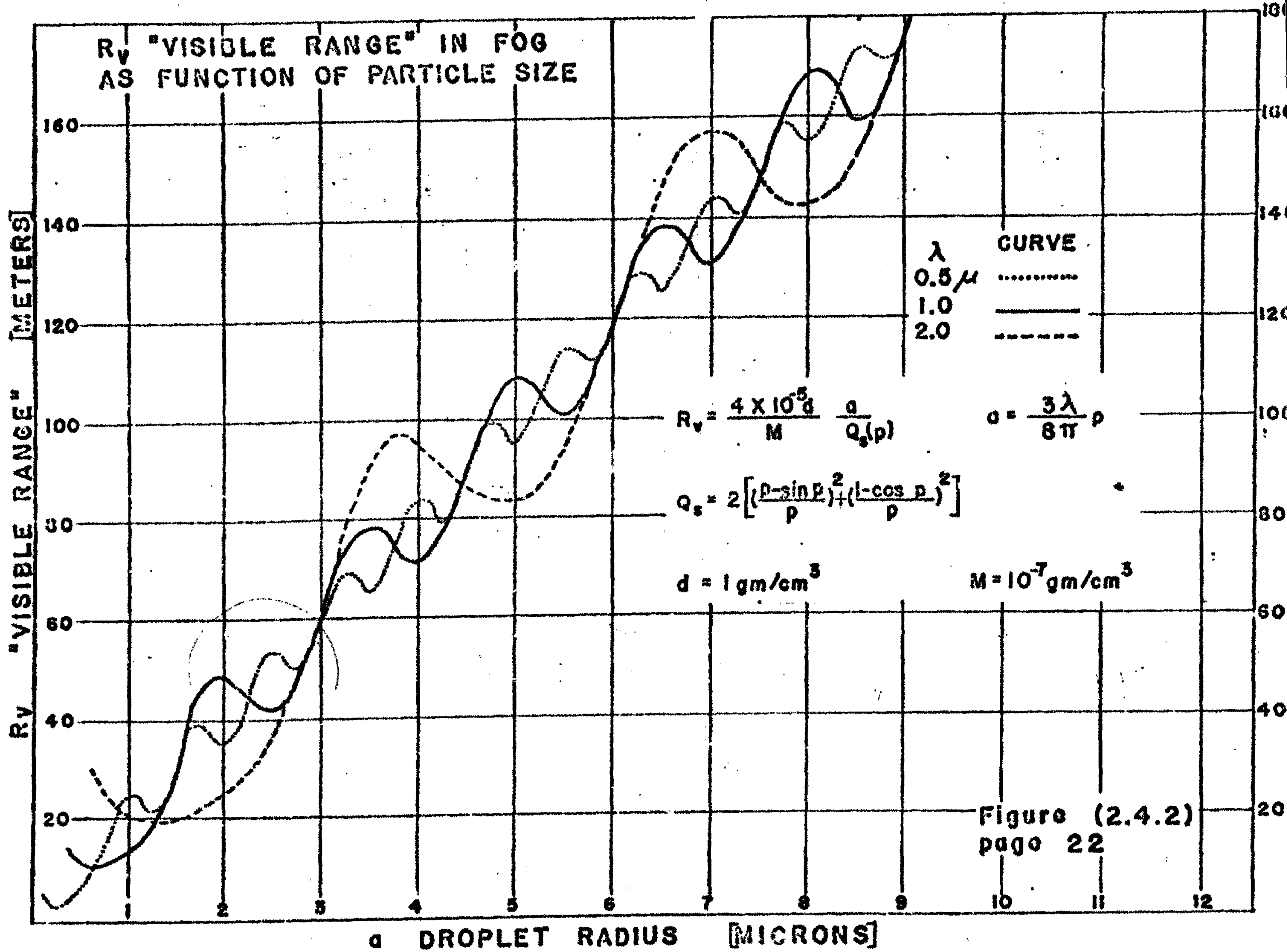


Figure (2.4.2)
page 22

$$\frac{4d}{3M} \frac{a}{Q_s} = 1.33 \times 10^5 \frac{a}{Q_s} \qquad (2.4.15)$$

is the distance in which a beam would weaken by a factor e due to scattering by water vapour in an atmosphere containing $M = 10^{-5}$ g/cc of liquid water of uniform drop diameter 2a. A distance of three times as much is known in meteorology as the "Visible Range". At that range, the contrast between a black object and the horizon haze would be below the threshold of discrimination for the normal human eye and not even a black object would be visible. A plot of the Visible Range

$$R_V = \frac{4d}{M} \frac{a}{Q_s} = 4 \times 10^5 \frac{a}{Q_s} \qquad (2.4.16)$$

is shown in Figure (2.4.2) as a function of drop radius a. For a wave length $\lambda = 1$ micron $= 10^{-4}$ cm and 2 micron diameter drops, i.e., $a = 10^{-4}$cm, we know from (2.4.7) that approximately $Q = 4$ so that in this example

$$R_V = 10^5 a = 10 \text{ cm} \qquad (2.4.17)$$

In such a fog, it would literally be "hard to see your hands before your face". Since fog and clouds normally have micron or larger particles which are comparable in Q_s, it must be that the liquid water content assumed ($M = 10^{-5}$ g/cc) is very much higher than is encountered. Even in a very dense fog or cloud, it is possible to see the wing tips of a plane or the

radiator cap of a car. The conclusions may be drawn, in any case, that clouds contain much less than 10^{-5} g/cc and that liquid water drops in the micron range are very effective in scattering light of wave length λ = 1 micron. It may also be concluded from (2.4.9) and (2.4.15) that R_V would go up linearly with particle size a for large drops in atmospheres of constant water content M. Thus, a mist with 1 mm drops would have about 2000 times greater Visible Range R_V than a 1 micron fog of the same liquid water content. Moreover, if the droplets were dispersed to much smaller size, say, 2a = 0.1 microns, then if we take note of (2.4.11) we see that for λ = 1 micron, Q_s would diminish with a^4 and R_V would increase with a^3. Such a fog or smoke would be even less effective than a mist with very large particles containing the same total amount of liquid water.

2.5 Transmission Laws for Thin Clouds

The discussion in the preceding paragraph showed that clouds can be very effective scatterers and that drop size is a very important parameter in such clouds. The comments made should not be taken to mean that because of the great effectiveness of water clouds, laser beams are useless over long paths. It is necessary first to discuss thin clouds, haze, mist and other climatic conditions more normal, or at least more common, over wide areas, than dense clouds. Actual clouds are polydisperse, that is, they contain particles distributed over a range of sizes. The total effectiveness is, therefore, not described by (2.4.6) but rather by

$$T(R) = e^{-\sigma R} \qquad (2.5.1)$$

where

$$\sigma = \pi \int_0^\infty N(a) Q_s(\rho) a^2 da \qquad (2.5.2)$$

The abundance distribution N(a) is best expressed in terms of cumulative liquid water content related to the M used in the preceding paragraph. The fraction F(a) of the mass M of liquid water suspended in unit volume in drop sizes up to a is defined as

$$F(a) = \frac{\frac{4\pi}{3}\int_0^a N(a) a^3 da}{\frac{4\pi}{3}\int_0^\infty N(a) a^3 da} \qquad (2.5.3)$$

The denominator is equal to the suspended water volume (or the mass of suspended liquid water per unit volume divided by the density of liquid water). This can be expressed as the thickness of the water layer that would be formed if all drops in a layer of atmosphere of unit thickness, say, one centimeter, were to coalesce into a sheet of liquid water. If we call τ the depth of liquid water precipitated or coalesced from 1 cm. layer of atmosphere, then the total thickness w (in centimeters) of precipitable water in a path R (in centimeters) would be

$$w = \frac{4\pi}{3} R \int_0^\infty N(a) a^3 da = R\tau \qquad (2.5.4)$$

Now we can see that (2.5.2) for the attenuation coefficient σ can be written

$$\sigma = \frac{3w}{4}\int_{0}^{\infty} \frac{Q_{s}(a)}{a}\,\frac{dF(a)}{da}\,da \qquad (2.5.5)$$

and consequently from (2.5.4) and (2.5.1)

$$T(R) = e^{-\sigma R} = e^{-\frac{3w}{4}\int_{0}^{\infty}\frac{Q_{s}(a)}{a}F'(a)\,da} \qquad (2.5.6)$$

This is a form of Beer's law for attenuation of monochromatic radiation. The exponent is proportional to the total equivalent depth of water w even though it is not an absorption in the liquid water but rather a scattering by droplets that is described by (2.5.6). The proportionality to w holds even if the M varies along the path R so long as F(a) is everywhere the same. Beer's law would not hold in an atmosphere that varied in particle size distribution from place to place. It is, in fact, an exceptional condition if Beer's law holds. One example of non-Beer attenuation has already been noted (2.3.6) in this paper and others will appear later. Equation (2.5.6) itself deviates from its simple form as soon as non-monochromatic radiation

is contemplated. Even laser beams which are notably monochromatic cannot be considered as exponential attenuators without further consideration.

The quantity dF/da in (2.5.5) is a mass distribution function. It describes the fraction of the water mass per unit size range in the size element da. Only a small fraction of the mass is in the form of very small droplets. Atmospheric aerosols will normally show a maximum in dF/da for one or more values of drop radius a. Small drops tend to evaporate in the presence of larger drops. Size is limited on the high side because settling rates increase rapidly with radius. It is advisable not to restrict discussion of distribution functions $F'(a)$ to simple powers of a because these would not permit a maximum. The form selected as interesting and general enough to discuss is a combination of two negative powers of a, namely

$$F'(a) = \frac{dF}{da} = \frac{A}{a^{n_1}} - \frac{B}{a^{n_2}} \qquad a > a_0 \qquad (2.5.7)$$

Here it is assumed that $n_2 > n_1 > 1$ to ensure that the curve has a maximum and that the cumulative distribution is finite, in fact

$$\int_{a_0}^{\infty} F'(a)da = 1 \qquad (2.5.8)$$

must be assumed according to the interpretation used here. The exclusion of values of a smaller than a_0 is necessary to avoid negative values of

$F'(a)$. The precise form of $F(a)$ for very small a is not important because of the minor contribution of small values to the total mass M and to the total scattering effectiveness Q_s. It is easy to verify that these properties are taken into account by substituting instead of (2.5.7) the more explicit equation

$$a > a_o \qquad F'(a) = \frac{(n_2 - 1)(n_1 - 1)}{a_o(n_2 - n_1)}\left[\left(\frac{a_o}{a}\right)^{n_1} - \left(\frac{a_o}{a}\right)^{n_2}\right] \qquad (2.5.9)$$

and its integral

$$F(a) \equiv \int_a^{\infty} F'(a)da = 1 - \left[\left(\frac{a_o}{a}\right)^{n_1-1} - \frac{a_oF'(a)}{n_2 - 1}\left(\frac{a}{a_o}\right)\right] \qquad (2.5.10)$$

This function does not have a negative slope anywhere. Its greatest slope is at the maximum of $F'(a)$ where $F''(a) = 0$. This occurs at a value of a called a_1 where

$$n_1\left(\frac{a_o}{a_1}\right)^{n_1} = n_1\left(\frac{a_1}{a_o}\right)^{-n_1} = n_2\left(\frac{a_1}{a_o}\right)^{-n_2} \qquad (2.5.11)$$

A second important radius a_2 is where the cumulative function $F(a_2)$ reaches half its ultimate value

$$F(a_2) = 1 - F(a_2) = \tfrac{1}{2} = \left[\left(\frac{a_o}{a_2}\right)^{n_1-1} - \frac{a_o F'(a_2)}{n_2 - 1}\left(\frac{a_2}{a_o}\right)\right]$$

$$= \left[\frac{n_2 - 1}{n_2 - n_1}\left(\frac{a_o}{a_2}\right)^{n_1-1} - \frac{n_1 - 1}{n_2 - n_1}\left(\frac{a_o}{a_2}\right)^{n_2-1}\right] \qquad (2.5.12)$$

Atmospheric aerosol distributions exist in considerable variety. It is normally not a question of predicting the function $F'(a)$ but rather a matter of adapting a formula to empirical size distributions. The importance of the subject has resulted in many experimental programs and has generated an extended bibliography, a start on which may be found in a recent article by Dobbins, Crocco and Glassman.(2.5.1) The laser may well help in the

(2.5.1) Measurement of Mean Particle Sizes of Sprays from Diffractively Scattered Light by R. A. Dobbins, L. Crocco and I. Glassman; AIAA Journal, Vol.1, No. 8, p. 1882 August 1963.

determination of particle size distributions, and so, it is useful to treat the question in some detail.

When observations of cloud, spray, or fog mass distributions are made and it is desired to try out the two power formula (2.5.9), the first problem is to find the two exponents n_1 and n_2. To plot the measurements and merely compare with curves for different sets of n_1, n_2 exponents is tedious and unsound. Far better and more efficient is to use the general features of the measured data in the manner presented herewith. A smoothed curve of the observations would show immediately what values to use for the quantities.

a_o - The smallest drop size to include.

a_1 - The drop size where $F'(a)$ is a maximum.

$F(a_1)$ - The cumulative fraction up to a_1.

$F'(a_1)$ - The mass abundance at a_1.

a_2 - The drop size corresponding to $F(a_2) = \frac{1}{2}$.

$F'(a_2)$ - The mass abundance at a_2.

These will serve to find n_1 and n_2. By specializing (2.5.9) and (2.5.10) for

$$a = a_1$$

and using equation (2.5.11), the exponential $(a_o/a_1)^{n_2}$ can be eliminated. Then n_2 can be eliminated between (2.5.9) and (2.5.10). Finally, the resulting transcendental equation for n_1 can be manipulated into the form

$$n_1 \left(\frac{a_o}{a_1}\right)^{n_1 - 1} = a_1 F'(a_1) + (1 - F(a_1)) \qquad (2.5.13)$$

which is easy to solve numerically. It yields two solutions. As could be predicted from (2.5.11), one solution gives n_1 and the other n_2 (n_2 n_1). To corroborate the solution, it is well to solve for n_2 from another equation which can be derived with the help of (2.5.9) and (2.5.10), namely

$$n_2 = 1 + \frac{a_1 F'(a)}{1 - F(a) - \left(\frac{a_o}{a}\right)^{n_1 - 1}} \qquad (2.5.14$$

It is also helpful sometimes to raise both sides of (2.5.13) to some higher power to improve the intersection which yields the numerical values sought by graphical means. It is noted that the parameters which appear in (2.5.1 and (2.5.14) - (when $a = a_2$) are just those listed previously. The procedu outlined is practical and avoids pitfalls which are easy to fall into when formal methods of curve fitting are used. Of particular danger are the influences of very high or very low values of a in prejudicing the curve fitting process.

An artificial example of the use of (2.5.13) to find n_1 and n_2 may be given by the following set of values for the necessary parameters. Suppose

$$a_1/a_o = 3/2 \; ; \qquad a_1 F'(a_1) = 4/9$$

$$1 - F(a_1) = 8/9$$

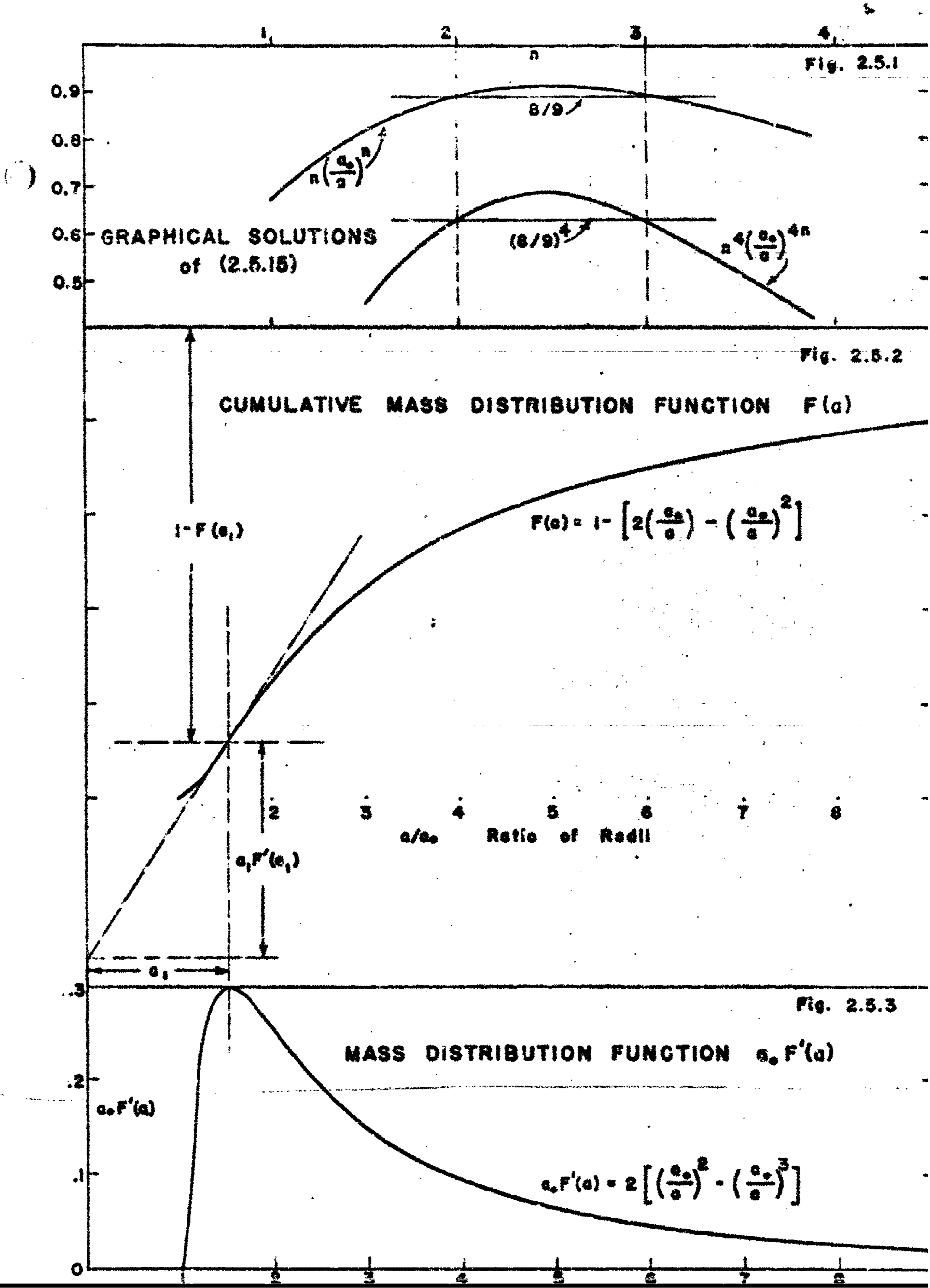


Fig. 2.5.1
n
GRAPHICAL SOLUTIONS
of (2.5.15)
8/9
$(8/9)^4$
$n\left(\frac{a_0}{a}\right)^n$
$n^4\left(\frac{a_0}{a}\right)^{4n}$
0.9
0.8
0.7
0.6
0.5
1
2
3
4
Fig. 2.5.2
CUMULATIVE MASS DISTRIBUTION FUNCTION F(a)
$F(a) = 1 - \left[2\left(\frac{a_0}{a}\right) - \left(\frac{a_0}{a}\right)^2\right]$
$1 - F(a_1)$
$a_1 F'(a_1)$
a_1
a/a₀ Ratio of Radii
Fig. 2.5.3
MASS DISTRIBUTION FUNCTION $a_0 F'(a)$
$a_0 F'(a)$
$a_0 F'(a) = 2\left[\left(\frac{a_0}{a}\right)^2 - \left(\frac{a_0}{a}\right)^3\right]$
.3
.2
.1
0
1
2
3
4
5
6
7
8

It follows that

$$n_1 (2/3)^{n_1 - 1} = 4/3 \tag{2.5.15}$$

This is easy to guess by inspection but if instead it is plotted out as in Figure (2.5.1), the roots turn out to be

$$\begin{aligned} n_1 &= 2 \\ n_2 &= 3 \end{aligned} \tag{2.5.16}$$

The steeper curve is a plot of the fourth power of the equ.ion just given, namely

$$n_1^4 (2/3)^{4n_1} = (8/9)^4 \tag{2.5.17}$$

Figure 2.5.2 is a graph of the corresponding cumulative mass distribution function F(a) while Figure (2.5.3) shows the mass distribution function $a_1F'(a)$ for the same values $n_1 = 2$; $n_2 = 3$.

The distribution shown in these figures is rather broad in the sense that particles ranging over a thousandfold in individual mass contribute significantly to the total mass of suspended water. Natural clouds occur which have much broader distributions and artificial fogs can be made with much narrower distributions. Of course, it is easy enough to describe distributions containing more than one maximum by summing terms similar to (2.5.9) with different pairs of exponents n_1 and n_2.

One merit of formula (2.5.9) is that, when applied in (2.5.6), it leads to an explicit transmission law for monochromatic radiation. In fact, the combination of (2.4.8), (2.5.5) and (2.5.9) yields

$$\sigma = \frac{3\tau}{4a_0}\,\frac{(n_2-1)(n_1-1)}{n_2-n_1}\int_{\rho_0}^{\infty}\frac{Q_s(\rho)}{\rho}\left[\left(\frac{\rho_0}{\rho}\right)^{n_1}-\left(\frac{\rho_0}{\rho}\right)^{n_2}\right]d\rho \qquad (2.5.18)$$

where in analogy with (2.4.8)

$$\rho_0 = \frac{4\pi a_0(m-1)}{\lambda} \qquad (2.5.19)$$

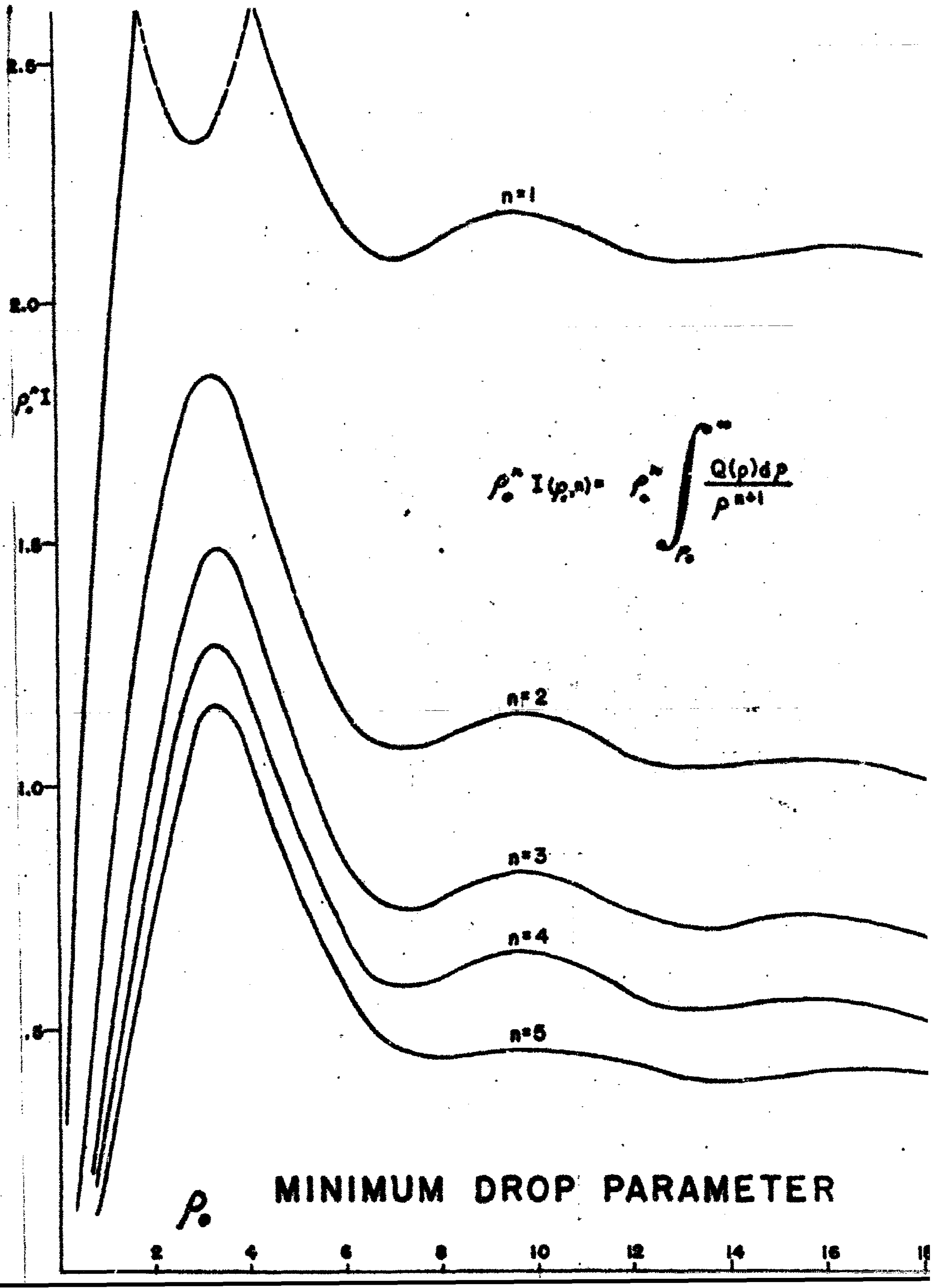
An abbreviation for (2.5.18) is

$$\sigma = \frac{3\tau}{4a_0}\,\frac{(n_2-1)(n_1-1)}{n_2-n_1}\left[\rho_0^{n_1} I(\rho_0, n_1) - \rho_0^{n_2} I(\rho_0, n_2)\right] \qquad (2.5.20)$$

where the symbol I is defined by

$$I(\rho_o, n) = \int_{\rho_o}^{\infty} \frac{Q_s(\rho)}{\rho^{n+1}} d\rho \qquad (2.5.21)$$

The drop scattering efficiency Q_s as written out in (2.4.7) consists of four terms composed of products of trigonometric functions and inverse powers of ρ . The integral (2.5.21) will thus also be made up of such elementary terms which would be easy to treat in any particular case. This does not suffice because all physically possible values of n are under consideration. (The empirically significant range of n is $1 < n < 5$ with the important values in the vicinity of $n = 3$. Of course, n need not be an integer.) The parameter ρ_o can be much smaller or much larger than unity and anything in between. The integrals in (2.5.21) may have singularities at $\rho = 0$ or infinity according to the value of n. Each value of n generates about $4(n + 1)$ terms before a manageable or negligible residual integral is obtained by the best methods so far attempted. The detailed discussion of the general result is too unwieldy and obscure. Instead a numerical integration was carried out and overall properties studied in the graphical form presented in Figure (2.5.4). The integrals $I(\rho_o, n)$ are inappropriate by themselves because of their very large values for small values of ρ_o and their very small values for large values of ρ_o. This trouble is avoided by discussing the product functions



2.5
2.0
1.5
1.0
.5
$\rho_0^n I$
n=1
n=2
n=3
n=4
n=5
$\rho_0^n I(\rho_0, n) = \rho_0^n \int_{\rho_0}^{\infty} \frac{Q(\rho)\,d\rho}{\rho^{n+1}}$
MINIMUM DROP PARAMETER
ρ_0
2
4
6
8
10
12
14
16
18

$$\rho_0^{\,n}\, I\,(\rho_0,\, n) \tag{2.5.22}$$

which p... y a part in the analysis of (2.5.21) and which actually appear in (2.5.20 The curves for $I(\rho_0, 1)$ and $I(\rho_0, 5)$ which are included in Figure ..5.4) are actually outside the permissible range of n. They are include: in order to reinforce the suggestions offered here for the reduction .f the functions (2.5.22).

The first observation to make on Figure (2.5.4) is the asymptotic behavio: at large abscissae for each value of n. This is an obvious consequenc: of equation (2.4.9) which permits (2.5.21) to be evaluated in the lim: . of large ρ_0.

$$I\,(\rho_0,\, n) \doteq \int_{\rho_0}^{\infty} \frac{2}{\rho^{n+1}}\, d\rho = \frac{2}{n\,\rho_0^{\,n}} \tag{2.5.23}$$

The curves in Figure (2.5.4) all seem to converge to zero for small va..ues of ρ_0. This is a convenience that comes from the factor $\rho_0^{\,n}$ in the product $\rho_0^{\,n}\, I\,(\rho_0,\, n)$. This product may be imagined to be expanded in a Taylor series about $\rho_0 = 0$ and then it is observed that the constant term is negligibly small. One or two terms of the Taylor series may suffice for small values of ρ_0 and these are the linear and quadratic terms in ρ_0.

It becomes apparent after some consideration that for small values of ρ_o up to unity or slightly greater, a limiting expression for $\rho_o^{\,n} I(\rho_o, n)$ in the following form is adequate for the present discussion.

$$\text{limit } \rho_o \to 0 \qquad \rho_o^{\,n} I(\rho_o, n) \approx \frac{1}{n}(c_1 \rho_o - c_2 \rho_o^{\,2}) \qquad (2.5.24)$$

Where c_1 and c_2 are constants whose sum is not very different from unity, Formula (2.5.24) has the desirable features that:- (a) it shows $\rho_o^{\,n} I(\rho_o, n)$ to go to zero when $\rho_o \to 0$ for all n; (b) it allows $\rho_o^{\,n} I(\rho_o, n)$ to reach about its asymptotic value 2/n at about $\rho_o = 1.5$ for all n; (c) it reproduces the curves for $\rho_o^{\,n} I(\rho_o, n)$ in Figure (2.5.4) at least qualitatively in the region of small values of ρ_o up to about $\rho_o = 1.5$; (d) it can be manipulated analytically as required in the present treatment of cloud transmission and so provides a practical approximation for (2.5.21).

The most interesting aspect of Figure (2.5.4) is that for values of ρ_o greater than about $\rho_o = 2$ the curves, in spite of their rather complicated shape, are separated from each other by remarkably constant displacements. Thus it is possible to say that $\rho_o^{\,n} I(\rho_o, n)$ can be represented as the sum of two parts

$$\rho_o^{\,n} I(\rho_o, n) = G_1(\rho_o) + G_2(\rho_o, n) \qquad (2.5.25)$$

where $G_1(\rho_o)$ is the same for all n and thus will cancel out in equation (2.5.20) which contains a difference of two product functions. The term $G_2(\rho_o, n)$ is seen to be almost constant for $\rho_o > 1.5$ and can be assigned the form (2.5.24) for $\rho_o < 1.5$. To be specific at the price of a small risk in numerical accuracy, the constants c_1 and c_2 in (2.5.24) could as well be accorded the values $c_1 = 0.8$; $c_2 = 0.4$ and then there is a continuous transition from the low ρ_o to the high ρ_o values of $G(\rho_o, n)$.

Now at last it is possible to work out the explicit law for turbid atmosphere extinction of a narrow laser beam. Equation (2.5.20) may be written

$$\sigma = \frac{3}{4}\frac{\tau}{a_o}\frac{(n_2 - 1)(n_1 - 1)}{n_2 - n_1}\left[\left(\frac{1}{n_1} - \frac{1}{n_2}\right) J(\rho_o)\right] \qquad (2.5.26)$$

where

$$J(\rho_o) = c_1\rho_o + c_2\rho_o^2 \qquad \rho_o < 1.5 \qquad (2.5.27)$$

$$J(\rho_o) = 2 \qquad \rho_o > 1.5 \qquad (2.5.28)$$

and approximately $c_1 \approx 0.8$; $c_2 \approx 0.4$.

It is easy and very useful to get rid of n_1 and n_2. First simplify

(2.5.26) to

$$\sigma = \frac{3}{4}\frac{\tau}{a_o}\frac{(n_2 - 1)(n_1 - 1)}{n_1\, n_2}\; J(\rho_o) \tag{2.5.29}$$

and then rise (2.5.13) and (2.5.14) to show that

$$\frac{(n_2 - 1)(n_1 - 1)}{n_1\, n_2} = \frac{a_1 F'(a_1)}{1 - F(a_1) + a_1 F'(a_1)}$$

It follows that

$$\sigma = \frac{3}{4}\;\frac{\tau}{a_o}\; J(\rho_o)\;\frac{1}{1 + \frac{1-F(a_1)}{a_1 F'(a_1)}} \tag{2.5.30}$$

and therefore the transmission law (2.5.6) in terms of water $w(= \tau R)$ is finally

$$\text{(II)} \qquad T(R) = e^{-\sigma R} = e^{-\frac{3}{4}\frac{w}{a_o}\frac{J(\rho_o)}{1 + \frac{1 - F(a_1)}{a_1 F'(a_1)}}} \tag{2.5.31}$$

A comparison with equations (2.4.12), (2.4.14) and (2.5.4) provides an interpretation of the first part of the exponential in (2.5.31), namely

$$\frac{3}{4}\frac{w}{a_o} = \frac{3}{4}\frac{\tau R}{a_o} = \frac{\tau}{\frac{4}{3}\pi a_o^3}\cdot \pi a_o^2 R = N(a_o)\,\pi a_o^2 R$$

If $w\ (= \tau R)$ grams of water were in the form of droplets of radius a_o, their total projected area would equal this same expression, $\frac{3}{4}\ \frac{w}{a_o}$. In the exponential in equation (2.4.14) the quantity Q_s would be interpreted as the efficiency of spheres of radius a in causing scattering and therefore extinction. Unit efficiency $Q_s = 1$ meant that a droplet of radius a removed from the incident beam as much light as would fall on an area πa^2. Similarly the quantity

$$Q_{eff} = \frac{J(\rho_o)}{1 + \dfrac{1 - F(a_1)}{a_1 F'(a_1)}} \qquad (2.5.32)$$

is interpreted as the efficiency in extinction of the actual cloud described

by $F(a)$ in (2.5.3) as compared with a cloud of the same water path w but composed of droplets all of radius a_o.

It is a most interesting property of Q_{eff} in (2.5.32) that it does not contain the parameters n_1 and n_2 of the special two power drop spectrum (2.5.10). It may therefore be expected that (2.5.31) will describe the extinction of clouds even when their drop size spectra differ considerably from the two power form treated in this paragraph so far. It is unnecessary in applying (2.5.31) to find the parameters n_1 and n_2. The quantities required can be taken directly from a plot of the empirical drop distribution function $F(a)$. In Figure (2.5.3), the radius a_1 is the abscissa of the maximum of $F'(a)$. The slope of $F(a)$ at that point is shown in Figure (2.5.2). The quantities $1 - F(a_1)$ and $a_1F'(a_1)$ are both indicated by vertical distances in this drawing. The ratio of these two distances on the graph is all that is needed to find the denominator of Q_{eff} in (2.5.32).

Measurement of laser beam transmission can be used in conjunction with equation (2.5.31) to give some valuable information on the drop spectrum of a cloud. Discussion of further properties of the transmission law is continued in the next paragraph.

2.6 Discussion of Cloud Transmission Laws

The cloud transmission law (2.5.31) has been put into a simple form only with the help of many drastic simplifications and approximations. The aim has been to arrive at a useful manageable expression, maintaining the physical characteristics and at least the qualitative numerical properties of the natural systems to be studied. Simple as it is in form, the law is hard to encompass, largely because it contains at least five essential parameters. These may be listed as:-

w - suspended liquid water path

a_o - minimum significant droplet radius

λ - wave length of transmitted light

$F(a_1)$ - cumulative mass-size distribution parameter

$a_1F'(a_1)$ - size abundance distribution parameter

It is not easy to discuss the formula clearly and concisely in a general sense and many errors of expression and interpretation have shown up in the literature in this connection. A few features of the transmission law will be listed.

(a) The first portion of the exponent, namely $3w/4a_o$, is the projected area of the total amount of suspended water in the path R, assuming it to be dispersed in droplets all of the same radius a_o. The water path w is usually given in millimeters. In that case, the drop radius a_o should also be expressed in millimeters. The rest of the exponential is therefore interpretable as the relative scattering effectiveness Q_{eff} of an average droplet as compared with a droplet of radius a_o.

(b) The ratio w/a_o is not a good parameter. Simple water path w is better and would help avoid certain misconceptions in the literature. This report, later on, will combine the a_o in the denominator with Q_{eff} in the form Q_{eff}/a_o. This is a property of the cloud size distribution and measure the effectiveness of a gram of water in a 1 cm^2 column of the particular cloud in the size distribution characterized by the quantities $F(a_1)$ and $a_1F'(a_1)$ which are easy to scale off in a plot of the mass size distribution function $F(a)$.

(c) Since all of the elements of Q_{eff} (also w and a_o) are positive, the largest possible value of Q_{eff} is 2 and the smallest is 0. The value 2 is approached for particles large compared with λ and a narrow size distribution. Broad distributions lead to smaller values of Q_{eff} even for large a_o/λ.

(d) The particular values $n_1 = 2$, $n_2 = 3$ used in the special example mentioned in the previous paragraph lead for large a_o/λ to the value

$$Q_{eff} = 2/3$$

This is most readily checked from equation (2.5.26).

(e) The scattering effectiveness of a cloud of particles larger than λ for one gram per cm^3 of suspended water varies inversely as the radius a_o of the minimum water drop. It has the form $3Q_{eff}R/4a_o$ where R is the path length in centimeters. Thus if the relative drop size distribution law remains unchanged, a reduction of the minimum drop size increases the light extinction, i.e., reduces the light transmitted, so long as the minimum drop size remains greater than the wave length of the light observed.

(f) When the minimum drop radius a_o is much smaller than λ so that $J(\rho_o)$ in (2.5.31) is proportional to ρ_o, then the scattering effectiveness per gram of water suspended in 1 cm^3 of cloud of a given distribution is independent of minimum drop size. The expression for σR then reduces to

$$\sigma R = \frac{3}{4} w \frac{0.8 \rho_o}{a_o\left(1 - \dfrac{1 - F(a_1)}{a_1 F'(a_1)}\right)} = \frac{3}{4} w \frac{3.2 \pi (m-1)}{1 - \dfrac{1 - F(a_1)}{a_1 F'(a_1)}} \frac{1}{\lambda} \qquad (2.6.1)$$

because $$\rho_o = 4 \pi (m-1) \frac{a_o}{\lambda}$$

If the distribution is narrow enough to say $1 - F(a_1) \ll a_1 F'(a_1)$, the expression (2.6.1) takes on a particularly simple and important form

$$\sigma R = 2.4\,\pi\,(m-1)\,\frac{w}{\lambda} \qquad (2.6.2)$$

Since $m - 1 = 0.33$ for visible light in water, this amounts to

$$\sigma R = 2.5\,\frac{w}{\lambda} \qquad (2.6.3)$$

This is an especially striking way of stating the effectiveness of a water cloud of small droplets in scattering light. It shows that a layer of water of thickness $w = \lambda$ would, if dispersed in a cloud of droplets smaller in radius than λ, attenuate a light beam by a factor of $e^{2.5} = 12$. For droplets of radius nearer λ, it would be appropriate to add a quadratic term to (2.6.3) that would about double the magnitude of the scattering effectiveness of a given cloud water layer. The cloud would then be more effective in scattering for shorter wave lengths. The dependence would be expressed in an exponent containing a λ^{-1} and a λ^{-2} term. If this were put into a single $\lambda^{-\alpha}$ term as frequently done in meteorology - following A. Ångstrom in 1929 - the exponent α would have to be between 1 and 2. This is actually reported to be the case for atmospheric haze and is called Ångstrom's law. The derivation of this law in the present report is quite different from those offered by van de Hulst (reference

(2.4.1), page 416 to 418). These are all subject to serious criticisms not applicable to the present report.

(g) A cloud or haze with a given drop size distribution F(a) will have a maximum monochromatic extinction coefficient σ for constant water path w at a wave length λ_{max} such that $\lambda_{max} = a_o$. This conclusion, too, provides rather a different interpretation from that usually accorded to chromatic effects in atmospheric attenuation. The practice has been to suppose that λ_{max} corresponded to the most abundant drop size rather than to the minimum significant drop size a_o.

All of the attributes listed have dealt with the scattering of visible light. If infrared or other wave lengths are considered, it would be necessary to remember that m is not equal to 1.33 for such light and sometimes complex values for refractive index m would have to be contemplated. This complication will not be attacked for the time being. It is more important at this moment to take up the question of non-monochromatic radiation. It might seem that such considerations would be unnecessary in connection with laser beams but this heretofore universal presumption will not always prove justifiable.

2.7 Non-Monochromatic Transmission

The wave length λ appears in the transmission law (2.5.31) only through the quantity $J(\rho_o)$ which is defined in equation (2.5.19), (2.5.27) and (2.5.28). For rain and aerosols whose droplets are all greater than λ, equation (2.5.28) holds and there is no apparent dependence on λ. The cloud scatters all wave lengths equally and is neutral or white. This corresponds to every day observation in that objects seen through heavy fog or clouds usually do not show much colour change. This result, together with others mentioned earlier, support the general characteristics of the

two power size distribution formulae for $F'(a)$ used to illustrate poly-dispersed aerosols in this report. By contrast, some of the distributions discussed on p. 193-4 of erference (2.4.1) seem to depend markedly on λ and would show the striking colour effects characteristic of artificial mono-disperse aerosols. These distributions, in the notation used in the present report, would have the forms

$$F'(a) = \begin{cases} 3\left(\frac{a}{a_o}\right)^2 & a < a_o \\ 0 & a > a_o \end{cases}$$

or

$$F'(a) = \begin{cases} 4\left(\frac{a}{a_o}\right)^3 & a < a_o \\ 0 & a > a_o \end{cases}$$

which do not simulate natural aerosol distributions. One distribution discussed by van de Hulst (p. 194) is equivalent to the form

$$F'(a) = \frac{1}{6}\left(\frac{a}{a_o}\right)^3 e^{-\frac{a}{a_o}}$$

This one is similar to that used in the present report in having a maximum

for some intermediate value of a. It leads to a grey or colourless cloud if the center of gravity of the size distribution is large enough. The other two distributions are truncated on the high side just where most of the water mass is. The truncation is bad only because it is so abrupt. The distribution used in this report could perhaps be helped by a suitable gradual truncation on the high side. This is not difficult to treat, but the discussion will be postponed. The subject of non-monochromatic light is treated in this paragraph in a preliminary fashion suitable only for thin cloud paths. The long paths required for actual laser beam studies will be elucidated in a sequel to this report.

When the particles go down to sizes much smaller than λ so that $\rho_0 << 1$, the aerosol is no longer neutral or gray. The function $J(\rho_0)$ has the form of (2.5.27); the linear term ρ_0 dominates and results in a λ^{-1} dependence on wave length as in (2.6.1) or (2.6.3). The long wave lengths are less scattered and the residual transmitted light takes on a reddish hue. This, too, is a familiar sight and again quite different from the varied colours of the sun often observed through artificial, monodisperse fogs.

The analytic treatment of a mixture of wave lengths is more troublesome than the method used to handle a mixture of drop sizes. The transmission law for a finite spectrum width rather than a single wave length is ordinarily expressed as an integral over wave length or frequency. Here it is convenient to use a variable proportional to frequency, i.e., the reciprocal of the wave length which is called the wave number, written $\bar{\nu}$ because it is proportional to frequency ν. The unit is reciprocal centimeters. The definition is

$$\bar{\nu} = \frac{1}{\lambda} = \frac{\nu}{c} \qquad (2.7.1)$$

where c is the velocity of light. Conventionally, the total transmission over a range in wave number $\bar{\nu}$ is written

$$T(R) = \int_0^\infty e^{-\sigma(\bar{\nu})R} K(\bar{\nu})\, d\bar{\nu} \qquad (2.7.2)$$

where $K(\bar{\nu})$ is defined as the fraction of the total beam energy which lies in a unit wave number interval between $\bar{\nu}$ and $\bar{\nu} + d\bar{\nu}$. There is no advantage for present illustrative purposes in using the complicated expressions for K( ) which can actually arise. It will suffice to make $K(\bar{\nu})$ constant over a narrow band from $\bar{\nu}_1$ to $\bar{\nu}_2$ and so write

$$K(\bar{\nu}) = \frac{1}{\bar{\nu}_2 - \bar{\nu}_1} \qquad \bar{\nu}_1 < \bar{\nu} < \bar{\nu}_2 \qquad (2.7.3)$$

$$K(\bar{\nu}) = 0 \qquad \bar{\nu} < \bar{\nu}_1 \text{ or } \bar{\nu} > \bar{\nu}_2 \qquad (2.7.4)$$

Thus

$$\int_0^\infty K(\bar{\nu})\, d\bar{\nu} = \int_{\bar{\nu}_1}^{\bar{\nu}_2} K(\nu)\, d\nu = 1 \qquad (2.7.5)$$

The transmission integral (2.7.2) can be worked out explicitly under much more general assumptions but to eliminate unnecessary algebra, the simple expression (2.6.3) will be discussed. Now the object is to find the transmission for a given water path w so the transmission will be considered a function of w rather than geometrical path R.

$$T(R) \rightarrow T(w) = \int_0^\infty e^{-2.5w\bar{\nu}} K(\bar{\nu})\, d\bar{\nu} = \frac{1}{\bar{\nu}_2 - \bar{\nu}_1} \int_{\bar{\nu}_1}^{\bar{\nu}_2} e^{-2.5w\bar{\nu}}\, d\bar{\nu} \qquad (2.7.6)$$

The monochromatic law which is often regarded as suitable for a narrow spectral range in which the absorption coefficient changes only slightly is called Beer's Law and is written for the present conditions

$$T_{\bar{\nu}_0}(w) = e^{-2.5w\bar{\nu}_0} \qquad (2.7.7)$$

where $\bar{\nu}_0$ is some frequency in the region $\bar{\nu}_1$ to $\bar{\nu}_2$. The explicit form of (2.7.6) is much less neat than (2.7.7), namely

$$T(w) = \frac{e^{-2.5\bar{\nu}_1 w}}{2.5w(\bar{\nu}_2 - \bar{\nu}_1)}\left(1 - e^{-2.5(\bar{\nu}_2 - \bar{\nu}_1)w}\right) \qquad (2.7.8)$$

Obviously it reduces to (2.7.7) in the limit of very short water paths when

$$2.5\,(\bar{\nu}_2 - \bar{\nu}_1)\,w \ll 1 \qquad (2.7.9)$$

It will be appreciated from the discussion after equation (2.6.3) that this is indeed a severe restriction because even a layer w equal to one wave length in thickness would fail to satisfy (2.7.9). For thicker paths, the ratio between (2.7.8) and (2.7.7) is

$$\frac{T(w)}{T\,\bar{\nu}_o(w)} = \frac{e^{2.5(\bar{\nu}_o - \bar{\nu}_1)w} - e^{-2.5(\bar{\nu}_2 - \bar{\nu}_o)w}}{2.5(\bar{\nu}_2 - \bar{\nu}_1)\,w} \qquad (2.7.10)$$

which is always close to unity so long as (2.7.9) holds. When w is large enough so that $2.5(\bar{\nu}_2 - \bar{\nu}_1)w \gg 1$, it is possible for $T(w)/T\,\bar{\nu}_o(w)$ to be either very small or very large compared with unity. If this situation is not recognized, a turbid atmosphere could appear much better or much poorer in the attenuation of a finite spectral band than expected. This is especially serious when complicated expressions for $J(\rho_o)$ or $K(\bar{\nu})$ arise. Molecular absorption by atmospheric gases which will be treated next is particularly troublesome in this regard. Meanwhile, reservations are to be held on the transmission of non-monochromatic light in the atmosphere when the attenuation is great enough to reduce the primary beam by

orders of magnitude. These are in order because the monochromatic transmission laws are approximate. The problem is further complicated for thick scattering layers by the contribution of multiply scattered light which can be more intense than the attenuated primary beam. It will be necessary to return to this question later on for various reasons. Laser beams are less subject to misinterpretation in hazy atmospheres than are ordinary sources, but it is prudent to have misgivings on this score even for lasers, especially when they fluctuate in wave length with time and emit a number of separate spectral lines or bands.

2.8 Water Vapour Absorption

The most interesting atmospheric effects on presently known laser beams are those due to the fine vibration-rotation line absorption in the carbon dioxide and water vapour of the atmosphere. The lines occur with irregularly varying intensity and abundance throughout the near infrared and even the reddish part of the visible spectrum. Emphasis will be placed on water vapour to begin with, because it is the more difficult and more important constituent to discuss as well as being the better known. There is a program at the National Bureau of Standards Laboratory in Boulder, Colorado for the detailed description of the water vapour absorption spectrum in the near infrared. An appreciation of the spectroscopic aspects of the problem and the extent to which they have been mastered may be obtained from recent publications on the 2.7 micron water band.[2.8.1],[2.8.2] There

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[2.8.1] D. M. Gates, R. F. Calfee, D. W. Hansen and W. S. Benedict, "Line Positions, Strengths, and Half-Widths for Water Vapor Bands ν_1, $2\nu_2$ and ν_3 in the Interval 2857 to 4444 cm^{-1}", National Bureau of Standards Monograph No. 71 (1963).

[2.8.2] David M. Gates, Robert F. Calfee, and David W. Hansen, "Computed Transmission Spectra for 2.7 - Micron H_2O Band." Applied Optics 2, 1117 (1963).

are more than four thousand lines. [illegible] lines listed in the [illegible] tables in the region from 2857 cm^{-1} ([illegible] microns) to 4444 cm^{-1} (2.25 microns). Thus the average spacing between lines is about 0.36 cm^{-1} while their half-width ranges from about 0.03 cm^{-1} to about 0.1 cm^{-1} at atmospheric pressure. The chance that a random wave length in this region from 2.25 to 3.5 microns will lie within a half-width of a line strong enough to be listed is therefore about 1/3. It is clear that for many laser beams, there will be overlapping with water vapour absorption lines. (Sometimes the absorption between lines can be important. Often the wings of a strong line hide many weak lines in the vicinity.)

Many of the lines listed by Gates, et al would be too weak to influence a laser beam perceptibly under ordinary atmospheric conditions. The situation is too complicated to formulate general rules for deciding which lines can be disregarded. It is much more practical to discuss a particular laser radiation and consider the way it is to be used. This will be done in the sequel to the present report. All that is required now perhaps, is to give an example wherein a typical water vapour line could play a part in laser communication.

The He-Xe gas laser reported by Faust, McFarlane, Patch and Garrett(2.8.3) provides a suitable illustration. The Xenon line at λ_{vac} = 2.6518 microns ($\bar{\nu}$ = 3771.0 cm^{-1}) is listed in Table III of reference (2.8.3) as a strong line in this laser. It comes in a strong part of the 2.7 micron water band discussed in the NBS reports listed. A narrow portion of the band from $\bar{\nu}$ = 3771 to $\bar{\nu}$ = 3773 is shown in detail in

(2.8.3) See A. Yariv and J. P. Gordon - "The Laser." Proc. IEEE, Vol. 52, p. 4, January 1963. This work of Faust et al is discussed in advance of publication on pages [illegible].

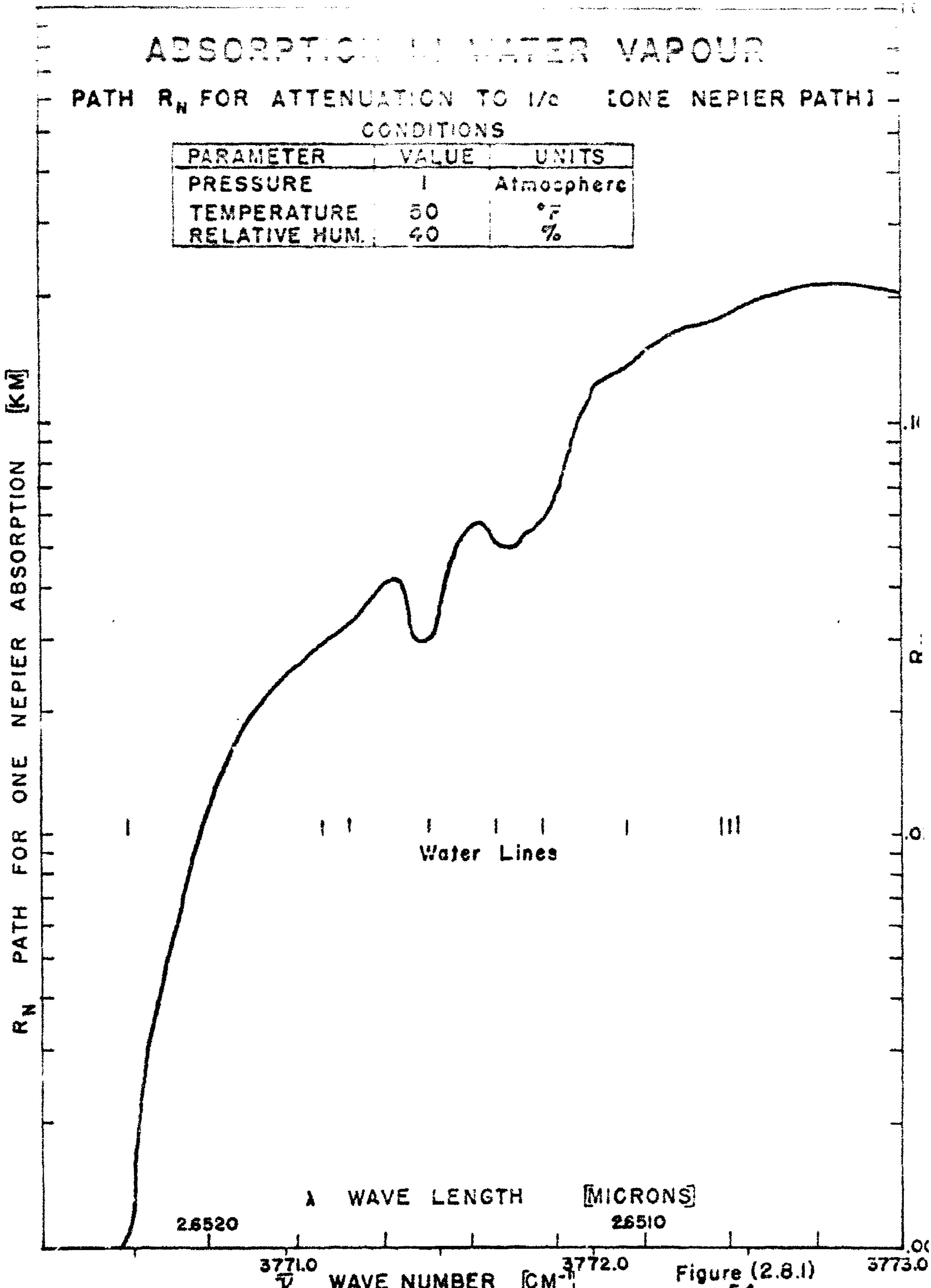
ABSORPTION IN WATER VAPOUR
PATH R_N FOR ATTENUATION TO 1/e [ONE NEPIER PATH]
CONDITIONS
PARAMETER | VALUE | UNITS
PRESSURE | 1 | Atmosphere
TEMPERATURE | 50 | °F
RELATIVE HUM. | 40 | %
R_N PATH FOR ONE NEPIER ABSORPTION [KM]
Water Lines
λ WAVE LENGTH [MICRONS]
2.6520
2.6510
3771.0
3772.0
3773.0
$\bar{\nu}$ WAVE NUMBER [CM^{-1}]
.10
.01
.00

Figure (2.8.1)

Figure (2.8.1). [illegible] a [illegible] moderate and some rather [illegible] this small [illegible] interval and the absorption coefficient runs from very high to moderate values in this interval. The positions of these lines are indicated by short, vertical lines located one-third the way up across Figure (2.8.1). The graph attempts to give a physical understanding of water vapour absorption in the near infrared. What is plotted is the distance in which a monochromatic light beam at each wave length across the spectral region would be attenuated by a factor e due to water vapour absorption if the sea level atmosphere was at 50°F and had a relative humidity of 40 percent. This distance is called the Nepier absorption path for these conditions.

The Nepier distance is computed with the help of line position, strength and half-width data tabulated in reference (2.8.1). The method will be indicated later, but first the figure will be discussed. The major contribution to the absorption throughout this interval comes from the strong lines at $\lambda = 3770.48$ and 3769.80 which are outside the interval. The weaker lines inside the interval are insignificant. The intermediate lines are of minor importance because nowhere do they modify by as much as 50 meters the distance at which a certain absorptance would be observed. (At higher elevations their influence would be more significant because the line shapes change with pressure. The strong lines would influence a narrower spectral region. Background absorption would be weaker and Nepier range greater. This makes the weak lines more important for two reasons.)

A laser beam line at 2.6518 microns would be attenuated by a factor e in traversing 25 meters. In 50 meters, it would be e^2 reduced and at 75 meters, $e^3 = 20$ fold absorbed. At 150 meters, the absorption

would b[illegible] 400 fold. Evidently a laser beam at this wave len[illegible] a long range communication implement. If the relative humidity drops to [illegible], the Napier distances cited all go up two fold and the statement just made holds as well as before. A change in temperature within the bounds of reason does not change the situation drastically. Only a change of height in the atmosphere would make a major difference. If the wave length would change merely from 2.6518 to, say, 2.6508 microns, the various Napier distances would increase by a factor of about six. Evidently it is necessary to be very specific about details in drawing conclusions about laser beam transmission because conditions change so rapidly. At another wave length, say, 3.5080 microns for example, where reference (2.8.3) lists a very strong laser line, the absorption effectiveness of water vapour is orders of magnitude weaker than in the midst of the 2.7 micron band which is so strong that no observable intensity of direct sunlight in this band reaches the earth. By contrast, the vicinity of the 3.5080 micron laser is free of observable water lines. The overall conclusion so far is that a further examination of water vapour transmission is worthwhile.

2.9 The Water Vapour Transmission Law

The attenuation of a laser beam is described in this report by a transmission function T(R) when the geometric path length R is emphasized and by T(w) when the depth of water w traversed is emphasiz[illegible]. This water depth w is defined as the thickness of the layer of liquid water that would be produced if the water in the system considered were condensed into a homogeneous liquid sheet. Previously the system consisted only of the liquid droplets suspended in the atmosphere. They attenuated the beam by scattering. Now attention is directed to the molecular water vapour alone. The water molecules absorb the light and thus attenuate the beam. If the

light is re-emitted subsequently, it is diffused in direction and still is lost from the primary beam whose transmission is studied here. The symbols for transmission laws used earlier in this report can serve for molecular absorption equally well. The absorption coefficient σ has to be specified in units appropriate to the multiplying factor R or w which appears in the formula for transmission. Thus

$$T(w) = e^{-\sigma w}$$

implies that σ is expressed in reciprocal centimeters of water depth if w is in centimeters of liquid water. A widely used form of molecular absorption law due to H. A. Lorentz is

$$\sigma(\bar{\nu}) = \frac{(S\alpha/\pi)}{\alpha^2 + (\bar{\nu} - \bar{\nu}_0)^2} \tag{2.9.1}$$

This applies to an absorber with an absorption maximum at wave number $\bar{\nu}_0$, a spectral "half-width" α ,corresponding to the wave number shift where the absorption coefficient has half the maximum value and a strength S defined by the integrated absorption coefficient so that

$$\int_{-\infty}^{\infty} \sigma(\bar{\nu})\, d(\bar{\nu} - \bar{\nu}_0) = S \tag{2.9.2}$$

The units for S, α, and $\bar{\nu}_i$ [illegible] reference [illegible] appro-
priate for present purposes in that if frequencies are expressed in cm^{-1}, and the half-life α and line strength S values taken from these tables, the results will be in reciprocal centimeters. If paths traversed by the light beams are expressed in centimeters of equivalent liquid water contained as vapour, the product σw will be dimensionless as required. There will be an absorption transition for water vapour at each wave number position $\bar{\nu}_i$ listed in the tables and a corresponding absorption term of the form (2.9.1) for each position. Thus at any spectral position $\bar{\nu}$, the absorption coefficient $\sigma(\bar{\nu})$ taking into account all absorption transitions would have the form

$$\sigma(\bar{\nu}) = \frac{1}{\pi} \sum_i \frac{S_i \alpha_i}{\alpha_i^2 + (\bar{\nu} - \bar{\nu}_i)^2} \qquad (2.9.3)$$

It is clear that terms for $\bar{\nu}_i$ remote enough from $\bar{\nu}$ will make no significant contribution to (2.9.3). The many closely spaced lines in the tables indicate the great numerical complexity of the problem and it is obvious that only a large digital computer would handle it satisfactorily. A form preferred over (2.9.1) in this report is

$$\sigma(\bar{\nu}) = \sum_i \frac{(S_i / \pi \alpha_i)}{1 + \left(\frac{\bar{\nu} - \bar{\nu}_i}{\alpha_i}\right)^2} \qquad (2.9.4)$$

This shows clearly how the [illegible] at the [illegible] center is given by

$$\frac{1}{\pi \alpha_i} \qquad (2.9.5)$$

and how α_i defines the half-width in such a way that when

$$\bar{\nu} - \bar{\nu}_i = \alpha_i$$

the local term in the absorption $\sigma(\bar{\nu})$ has dropped to one-half of its value at the center $\bar{\nu}_i$. It is a fortunate circumstance in getting a feeling for water absorption that while the strengths S_i tabulated for the 2.7 micron band vary from 10^{-3} to almost 10^{4}, the half-widths vary only about threefold from about 0.03 cm^{-1} to about 0.1 cm^{-1}. It is a fair estimate to take for all strong lines a rough mean value for all i

$$\alpha_i \doteq 0.07 \text{ cm}^{-1} \qquad (2.9.6)$$

It follows that at any position $\bar{\nu}$ the absorption contribution of distant lines (and even nearby ones) will be approximately

$$0.02 \frac{S_i}{(\bar{\nu} - \bar{\nu}_i)^2} \qquad (2.9.7)$$

Thus strong lines extend their influence over a spectral region roughly proportional to

$$\sqrt{S_i} \tag{2.9.8}$$

This is what explains the dominant trend of the Napier path curve shown in Figure (2.8.1). It also indicates that for distant communications, laser lines far away from strong water lines must be selected. The strong water lines are concentrated near band centers. Between bands, there are regions called windows where few lines are found and practically no strong lines occur. Lasers that emit in these windows are likely to be the ones that reach to great distances. The intensity of the laser radiation is less important than its absorption coefficient which appears in the exponent of the transmission law.

The considerations of water vapour absorption so far have presumed only monochromatic laser beams. When multiple discrete lines or continuous bands are emitted, it is necessary to sum or integrate over the spectrum. The summation is conventionally carried out over all frequencies or wave lengths, but a different procedure will be presented here. It has been developed to handle the problems of exponential attenuation which were alluded to in paragraph (2.7). There it was brought out that small changes in exponential absorption coefficients for composite radiations can make major differences in the composition of the radiation transmitted through long paths.

The absorption coefficient $\sigma(\nu)$ in (2.9.4) shows how the absorption at a single frequency ν is influenced by selective water

vapour absorption at various frequencies $\bar{\nu}_i$. If there are different frequency components $\bar{\nu}_b$ of the laser in the proportion $K_b(\bar{\nu})$ so that

$$\sum_b K_b(\bar{\nu}) = 1 \qquad (2.9.9)$$

then each component has its own absorption coefficient $\sigma_b(\bar{\nu})$ and the transmission at water path w has the form

$$T(w) = \sum_b K_b(\bar{\nu})\, e^{-\sigma_b(\bar{\nu})w} \qquad (2.9.10)$$

where each $\sigma_b(\bar{\nu}_b)$ is described by its own equation instead of (2.9.4)

$$\sigma_b(\bar{\nu}) = \sum_i \frac{S_i/\pi\alpha_i}{1+\left(\frac{\bar{\nu}_b-\nu_i}{\alpha_i}\right)^2} \qquad (2.9.11)$$

Each laser frequency $\bar{\nu}_b$ has its own Nepier water path w_{Nb} which is the reciprocal of $\sigma_b(\bar{\nu})$. If the relative strength of the component of wave number $\bar{\nu}_b$ is written $K_b(\sigma)$ this emphasizes the fact that the component has a characteristic σ_b as well as a frequency or wave number $\bar{\nu}_b$. The expression (2.9.10) may therefore be regarded as a summation over absorption coefficients rather than a summation over frequencies. If

moreover a laser emits a continuous band, the spectral distribution in the interval d $\bar{\nu}_b$ could be described by $K(\bar{\nu}_b)d\bar{\nu}_b$ or equally well by

$$K_b(\sigma)\left(\frac{d\bar{\nu}}{d\sigma}\right)_b d\sigma \qquad (2.9.12)$$

where

$$\left(\frac{d\sigma}{d\bar{\nu}}\right)_b \qquad (2.9.13)$$

is the slope of the absorption coefficient curve in the particular band or region denoted by subscript b. If the absorption coefficient curve is continued across the spectrum instead of covering a single narrow band of frequencies, there will in general be many different places where the same absorption coefficient σ would arise. Thus the summation would include many terms each with the same exponential factor. The general transmission law for the entire laser beam including many lines or continuous bands can now be written

$$T(w) = \int_0^\infty e^{-\sigma w} \sum_b K_b(\sigma)\left(\frac{d\bar{\nu}}{d\sigma}\right)_b d\sigma \qquad (2.9.14)$$

The summation in (2.9.14) will be called the expectation of σ or the

distribution and will be assigned the symbol $p(\sigma)$.

$$p(\sigma) = \sum_b K_b(\sigma)\left(\frac{d\bar{\nu}}{d\sigma}\right)_b \qquad (2.9.15)$$

It is a joint property of the laser beam through $K_b(\sigma)$ and of the absorbing medium characterized by $(d\sigma/d\bar{\nu})_b$. The fact that $\bar{\nu}$ is not a unique function of σ will not be troublesome if the regions b in the laser spectrum are defined in terms of intervals between successive maxima in the absorption coefficient curve $\sigma(\bar{\nu}_{\chi})$. The singularities at the points where

$$\left(\frac{d\sigma}{d\bar{\nu}}\right)_b = 0$$

will not be serious because they make infinitesimal contributions to the integral (2.9.14). (It would be out of place to dwell on the many interesting mathematical properties of the present argument. They might obscure the physical conception which is quite simple when once perceived.)

The distribution $p(\sigma)d\sigma$ measures the fraction of the total initial laser beam in the absorption coefficient interval $d\sigma$. The final transmission law is now seen to be the Laplace transform of the absorption coefficient distribution

III $$T(w) = \int_0^\infty e^{-\sigma w}\, p(\sigma)\, d\sigma \qquad (2.9.16)$$

It would, of course, have been just as easy to define absorption coefficients in units appropriate to geometrical paths R instead of water paths ω . Then that transmission law would be written

$$\text{III}' \qquad T(R) = \int_0^\infty e^{-\sigma R} \, p(\sigma) \, d\sigma \qquad (2.9.17)$$

This would be just as convenient as (2.9.16) but in some discussions would describe conditions in a geometrical way easy to visualize. The III or III' are suitable for extinction in scattering media as well as for molecular absorption. They transform an observable function $p(\sigma)$ of the absorption or scattering coefficient to another observable function $T(R)$ or $T(w)$ of the path thickness. The exponential expression is not an approximate or empirical quantity but a definite operator. The uncertainties in absorption coefficient which were discussed in paragraph (2.7) are removed from the sensitive exponential operator to the pliable functions $p(\sigma)$. This is more than a mere convenience as will appear in the sequel to this report.

3. Comments, Conclusions and Summary

3.1 Illustrative Computations

Transmission formulae III and III' in the last paragraph are simple enough when a suitable form for the absorption coefficient distribution $p(\sigma)$ is at hand. The problem for the complicated absorption spectra in the atmosphere is to find a workable expression for $p(\sigma)$. The presentation used here is novel enough to justify an illustration of how equation (2.9.17) works out when absorption lines of Lorentz shape (2.9.1) occur. Thus it is supposed that only one term counts in (2.9.4) and furthermore suppose that for all frequencies $\bar{\nu}$ of interest

$$\bar{\nu} - \bar{\nu} \gg \alpha \qquad (3.1.1)$$

so that it is permissible to write approximately

$$\sigma \doteq \frac{\alpha S/\pi}{(\bar{\nu}-\bar{\nu}_0)^2} \qquad (3.1.2)$$

and therefore to use in (2.9.15)

$$\frac{d(\bar{\nu}-\bar{\nu}_0)}{d\sigma} = -\frac{\alpha S}{\pi}\,\sigma^{-3/2} \qquad (3.1.3)$$

so that since only one term is ne d

$$p(\sigma)d\sigma = \frac{\alpha S}{2\pi} \frac{K(\sigma)}{\sigma^{3/2}} d\sigma \tag{3.1.4}$$

The function $K(\sigma)$ was defined as the incident light spectral distribution in terms of absorption coefficient σ. The example chosen now is for an incident spectrum which is gaussian about the absorption line whose center is at frequency or wave member $\bar{\nu}_o$, that is assume

$$K(\bar{\nu}) \sim e^{-a(\bar{\nu} - \bar{\nu}_o)^2} \sim e^{-a(\alpha^2 + (\bar{\nu} - \bar{\nu}_o)^2)}$$

Then $K(\sigma)$ would have the form

$$K(\sigma) \sim e^{-\frac{aS\alpha}{\pi}\frac{1}{\sigma}} \tag{3.1.5}$$

and the transmission law (2.9.17) is

$$T(R) = K_o \int_0^{\infty} e^{-\sigma R} \frac{e^{-\frac{aS\alpha}{\pi\sigma}}}{\sigma^{3/2}} \, d\sigma \qquad (3.1.6)$$

where K_o is simply a normalizing factor to be chosen so that

$$T(0) = 1$$

and the transmission is 100 percent for zero path ($R = 0$). The expression (3.1.6) is a standard Laplace transform found, for example, in the tables of the Handbook of Chemistry and Physics. It is seen that for any path R the transmission is

$$T(R) = e^{-2\sqrt{\frac{aS\alpha}{\pi} R}} \qquad (3.1.7)$$

A most interesting feature in (3.1.7) is that the path R appears under a radical in the exponential. This so-called square root law is observed for absorbers with line spectra provided they are in a special thickness range. They must not be so thin that the line centers play a part. The light near the line centers is not properly treated in equation (3.1.2). They must also be thin enough so that the light very far from the line center passes through the absorber almost unattenuated. The Lorentz line shape formula on which this computation depends does not hold far from the

line centers. [illegible] which it could emit itself might be of interest in passing laser radiation through another laser. This may not seem to be the kind of subject to treat in this report on communications and so an apparently quite different example is offered.

Consider a laser beam covering a narrow spectral range between $\bar{\nu}_1$ and $\bar{\nu}_2$

$$\bar{\nu}_0 < \bar{\nu}_1 < \bar{\nu} < \bar{\nu}_2 \qquad (3.1.8)$$

and of uniform spectral strength within this interval so that $K(\bar{\nu})$ or $K(\sigma)$ in (2.9.15) would be constant within (3.1.8) $K_g(\sigma) = \frac{1}{\bar{\nu}_2 - \bar{\nu}_1}$ and zero outside. Let the region (3.1.8) lie fairly close to one particular water vapour line and remote enough from all others so that only one term need be considered in (2.9.15). The assumption will be made again that water vapour has Lorentz line absorption and that approximately (3.1.2) and (3.1.3) hold. Now with these assumptions, the transmission (2.9.16) can be written

$$T(w) = \frac{1}{\bar{\nu}_2 - \bar{\nu}_1} \int_{\sigma_1}^{\sigma_2} e^{-\sigma w} \frac{d(\bar{\nu} - \bar{\nu}_0)}{d\sigma} \, d\sigma \qquad (3.1.9)$$

where by (3.1.2) the limits are defined by

$$\sigma_1 = \frac{\alpha S}{\pi(\bar{\nu}_1 - \bar{\nu}_0)^2} \quad ; \quad \sigma_2 = \frac{\alpha S}{\pi(\bar{\nu}_2 - \bar{\nu}_1)^2}$$

Integration by parts shows

$$T(\omega) = \frac{(\bar{\nu}_2 - \bar{\nu}_0)e^{-\sigma_2 \omega} - (\bar{\nu}_1 - \bar{\nu}_0)e^{-\sigma_1 \omega}}{\bar{\nu}_2 - \bar{\nu}_1} + \omega\sqrt{\frac{\alpha S}{\pi}} \int_{\sigma_1}^{\sigma_2} \frac{e^{-\sigma\omega}\, d\sigma}{\sqrt{\sigma}} \qquad (3.1.10)$$

The integral is easy to work out or find in a table of Laplace transforms.

The result is

$$T(\omega) = \frac{\bar{\nu}_2 - \bar{\nu}_0}{\bar{\nu}_2 - \bar{\nu}_1}\left(e^{-\sigma_2 \omega} - \sqrt{\pi\sigma_2\omega}\cdot \operatorname{erfc}\sqrt{\sigma_2\omega}\right)$$

$$- \frac{\bar{\nu}_1 - \bar{\nu}_0}{\bar{\nu}_2 - \bar{\nu}_1}\left(e^{-\sigma_1 \omega} - \sqrt{\pi\sigma_1\omega}\cdot \operatorname{erfc}\sqrt{\sigma_1\omega}\right) \qquad (3.1.11)$$

To get the signs right in this expression, it is necessary to be careful and note that in the light of (3.1.8)

$$\sigma_1 > \sigma_2$$

and so the integral in (3.1.10) is inverted relative to the normal direction of integration.

An interesting and important variant of (3.1.11) is obtained when $\bar{\nu}_1$ coincides with the absorption line center $\bar{\nu}_o$. Then (3.1.11) simplifies to a single term

$$T(w) = (e^{-\sigma_2 w} - \sqrt{\pi \sigma_2 w} \cdot \operatorname{erfc} \sqrt{\sigma_2 w}) \qquad (3.1.12)$$

This is regarded as the fundamental transmission formula for an absorber with narrow Lorentz lines. The situation is actually very similar to that in the first example described by (3.1.6). In both examples, the spectral intensity is essentially a step function which is zero for small σ and constant for large σ. The exponential square root law (3.1.7) seems quite different from (3.1.12) but when (3.1.12) is calculated out it is for all practical purposes the same as (3.1.7). This important fact is in good agreement with observations on the absorption of infrared light in moderate water paths - up to about 1 millimeter of precipitable water. The great significance of (3.1.12) arises because it can be shown to apply across the entire infrared absorption spectrum of the atmosphere. A narrow laser beam absorption can be derived by taking a weighted difference between

the ... absorption coefficients σ_1 and σ_2 corresponding to the boundary wave numbers $\bar{\nu}_1$ and $\bar{\nu}_2$.

There are many details to take up before (3.1.12) can provide an entirely acceptable formulation of narrow, multiple line transmission, but it is best to postpone these details until a specific requirement for them arises.

3.2 Broad Laser Spectra

It is characteristic of coherent beams that their spectral intensities vary rapidly with wave length. Their spectral width may be narrower than the line width α characteristic of water vapour where $\alpha \sim 0.1\ cm^{-1}$ but often - especially for semi-conductor lasers - there is evidence of much broader emissions. Most of the experimental work reported in the literature has so far depended on equipment incapable of resolving better than 10 wave numbers which is several times the spread of Figure (2.8.1) overlapping more than ten water lines. In such a broad region, the Nepier path could vary by orders of magnitude as it does in Figure (2.8.1). Without knowing the fine details of the laser emission, it is not feasible to give a close estimate of transmission in such a situation - although limiting values could be stated. On the other hand, an observation of the transmission of a laser beam might provide a relatively easy way of finding out some details of its spectral distribution. If, for example, a laser line near λ = 3.6518 microns in wave length had a Nepier path less than 30 meters of air under the conditions of Figure (2.8.1), then it is clear from the figure that there could not be as much as 30% of the beam in the wave length region shorter than λ = 3.651... If the Nepier path were longer than 10 meters, there could hardly be as much as half the radiation in the ...

wave length region longer than [illegible]. [illegible] obser-
vation of the atmospheric transmission in a particular experiment would
give more detailed information on the laser beam spectrum.

3.3 Time Varying Spectra

The comments on broad spectra would apply as a time average if the laser radiated narrow lines which shifted in wave length with time. If the transmission were monitored in time, it might be feasible to conclude how wave length changed by means of transmission curves such as that in Figure (2.8.1).

3.4 Temperature and Pressure Effects

The laser system itself is, of course, subject to change due to environmental effects. The observed shifts in wave length or spectral distribution can easily bring the light into or out of a region of strong absorption because the absorption coefficient $\alpha(\lambda)$ for water vapour or CO_2 is such a sensitive function of frequency. It is to be expected that solid state infrared laser beam intensity, for example, will show complicated drifts when observed through long air paths. Small spectral shifts might scarcely influence aerosol attenuation but could produce violent changes in vapour absorption. This may be a mechanism for distinguishing between the two kinds of attenuation. The subject is worthy of numerical examination when particular laser beams are to be tried out over long outdoor paths.

The primary topic for this paragraph is, however, not the changes in the laser system but rather the changes with temperature and pressure in the atmospheric absorber. These arise because the Lorentz shape of the absorption lines is due to collisions suffered by the absorbing molecules. The number of collisions will vary with temperature and pressure, and so,

[illegible] vary in [illegible] and in [illegible] absorption coefficients σ. The appro-priate [illegible] for water [illegible] in air are given in Reference (2.5.2) [illegible] page [illegible].

$$S = S_o \left(\frac{T_o}{T}\right)^{3/2} \cdot e^{-\left[\frac{E''}{kT_o}\,\frac{T_o}{T} - 1\right]} \tag{3.4.1}$$

and

$$\alpha = \alpha_o \left(\frac{P}{P_o}\right)\left(\frac{T_o}{T}\right)^{0.62} \tag{3.4.2}$$

where P is total gas pressure and E''/kT_o is the Boltzman exponent for the energy of the lower state for the line in question. The term values E'' are listed in Reference (2.8.1). The pressure dependance is particularly important in the upper atmosphere, but for present purposes, interest attaches mainly to relatively small changes $\Delta T = T - T_o$ and $\Delta P = P - P_o$ observed in temperature and pressure at sea level. The treatment in this report has included the parameters S and α as a product in expressions such as (2.9.1) for absorption coefficients σ. The variation of transmission for small changes in temperature and pressure can be obtained by expanding (3.4.1) and (3.4.2) and discarding second order and higher terms. The result can be expressed in the form

$$\sigma = \sigma_0 [1 - \ldots \text{[illegible]}] \qquad (\ldots)$$

where the term values E" are in wave n[illegible] (cm^{-1}) [illegible] of Reference (2.3.1). At ordinary, outdoor sea level temperatures, k[illegible] is not far from 200 cm^{-1}. The [illegible] lines are likely to arise from states with small E" and [illegible] the temperature and pressure coefficients of absorption are likely to [illegible] negative for l[illegible] beams near str[illegible]. For beams near weak li[illegible], th[illegible] temperature coefficients may well [illegible] some laser beams could become stronger and others weaker as a resul[illegible] of a temperature change over the same air path.

3.5 Magnetic and Electric Fields

The fields acting on molecules [illegible]read over long paths in the open atmosphere are not strong enough to produ[illegible] effects significant for laser beam attenuation. Of course, in the laser system itself, electr[illegible]gnetic fields are effective. They cause changes in spectral distribution in the laser source which lead to changes in beam attenuation. Polarization, modulation, angular deviation, and coherence effects associated with electromagnetic fields also play important parts in laser beam communications but these are not treated here. When they are important, they would have to be considered separately - beyond the treatment of primary attenuation in this report.

The electromagnetic effects possible at the source are so vari[illegible] that it does not seem useful to discuss them in a general way. As an example

consider the formation of higher frequency spectra by non-linear action of laser beam fields. New photons produced in high fields have quite different attenuation properties from the primary beam. These combination photons depend on the square or higher powers of the laser beam field, and so, a slight modulation of the laser system could produce a marked modulation of the second or third order radiation. This could be the basis of a novel signaling system. In the course of this study, it will be necessary to compare this system with the straightforward intensity modulation of a laser beam with no change in wave length and also with methods where slight wave length changes are brought about by magnetic fields. Such slight wave length changes could shift the laser beam into and out of an atmospheric absorption line and thus modulate the light reaching out some distance from the source while at close range no effect would be apparent. The different systems are to be compared, not only in the ease and degree of modulation attained, but also in the way the modulation influences the transmission of the beams under various atmospheric conditions.

3. Operating Conditions

The vapour line absorption, the scattering by aerosols and the degradation of laser beams by random optical density gradients in the atmosphere have been treated separately but they must be regarded as concurrent in their action. The atmosphere is never free of any one of these influences though their relative importance changes over extreme ranges with environmental conditions. The great variability complicates the communications equipment more than the transmission laws. It becomes necessary to design the generating, transmitting and sensing elements as coordinated parts of a whole system in order to achieve optimum and reliable communications. The discussion in this report was intended merely to supply background for the

overall system design and to help provide realistic specifications and numerical values for some of the components of communication systems.

The energy requirements at the source, the optics of the collimating and receiving systems, and the performance required of the sensors and recording elements are very much dependent on atmospheric conditions, the physics and geometry of the light path as well as the kind of information to be transmitted. To be specific, but without any notion of being precise, the first laser application to communications will be pictured as a walkie-talkie type of use. The exponential character of the atmospheric attenuation laws means that there is no practical way of gaining much in range by increasing laser power. It appears better to give up the thought of reaching the horizon and to strive for portability and convenience instead. Even now there are fuel cell power plants weighing no more than 50 lbs. with a power output of at least 1 kilowatt. The laser beam is intrinsically small in cross-section, perhaps of the order of a centimeter , and even the receiving optics could well be limited to less than 10 centimeters in diameter. The system might well be truly "line of sight" and its virtues would not be wide coverage but rather extreme selectivity. At a distance of, say, one mile (1.6 kilometers), the entire beam width at the receiver might be less than one foot in diameter. Thus one man in a group could be singled out for one message and a second man could get a different message, provided both could be sighted in a telescope. The auxilliary equipment for such a system is already known. Nearby ships, boats, or planes would be accessible, and sometimes telephone communications from one part of the deck of a large ship to another could be carried out more conveniently by laser beam than otherwise. It should be noted finally that a narrow beam can easily be spread at will either in two dimensions or in one. Moreover, the beam can

be oscillated or scanned to cover several receivers or to stabilize to a fixed direction even from a moving vehicle. The compactness and light weight of the laser is here and in many respects as important as the coherence of the beam.

3.7 Range Limitations

The laser has been pictured in the previous paragraph as a rather short range signalling device. This does not mean it can never attain long range in the lower atmosphere. The limiting range is assessed most convincingly by reference to observational material. The valuable atmospheric absorption atlas of the Naval Research Laboratory(3.7.1) is very useful here. A ten statute mile path, 50 feet over Chesapeake Bay, was used by NRL to produce rather high resolution absorption spectra in an atmosphere at an average of 75°F. and 65% relative humidity. The vapour path thus contained the equivalent of about 23 centimeters of precipitable water. The visible and near infra spectral regions covered by this atlas contains hundreds of lines of all intensities from barely distinguishable from the background to fully absorbing. The background illumination was simply a 1000 watt projection lamp at the focus of a 60 inch searchlight. It was strong enough between absorption lines to give a photographic exposure at a dispersion of about 8 min/Å at the focus of a 20 foot focus f:13 mirror. The exposures ran from 5 to 60 minutes. A monochromatic signal ten times weaker than the background illumination between lines would have been readily discerniblo. Thus aerosol scattering can be small enough so that laser signals reach out to the horizon under some circumstances. It is obvious, of course, that the visible light laser can reach at least as far as any object can be

(3.7.1) J. A. Curcio and G. L. Knestrick; An Atlas of the Absorption of the Atmosphere from 5400 to 8520 Å. NRL Report 4601, August 23, 1955.

seen under the same atmospheric conditions. It is a particularly strong source and what it lacks in initial beam width can be made up for signalling purposes, with the help of collimating optics. If a mountain or a horizon can be seen, so can a laser. The rate of information transmitted is another matter - that would be determined by the energy in the beam and the sensitivity and other properties of the receiver. The NRL Atlas is a particularly good source of information on the atmospheric absorption lines likely to interfere with long range laser communications in the visible and near infrared. It shows at once that while simple signalling with laser beams in clear weather is possible practically anywhere in the visible spectrum, there are many wave lengths where the absorption due to oxygen or water vapour would be a serious drawback at distances above one kilometer.

The infrared is different and perhaps more interesting and that is why this report has emphasized the infrared. The scattering by aerosols is much less as brought out in equation (2.6.3) and many other equations developed in this report. The scattering exponent σ for a given water path goes down inversely with a power of λ between one and two (Ångstrom's Law par. 2.6). Thus if a certain distance can be traversed in the visible, scattering may well be ten times less at, say, 10 microns. For turbid atmospheres, the infrared is more effective and the sensors available show better performance. The line absorption, on the other hand, is very much stronger than in the visible.

There are, however, regions in the infrared where line absorption is particularly weak. These so-called 'windows' were discussed most recently by Bignell, Saiedy and Sheppard[3.7.2] who made extensive measurements of

[3.7.2] Bignell, K., Saiedy, F. and Sheppard, P. A.; On the Atmospheric Infrared Continuum. JOSA, 53, p. 466. (1963).

atmospheric attenuation between 8 and 13 microns. They find that aerosols are frequently much less important in the infrared windows than the continuous absorption due to the wings of the strong water vapour bands on either side of a window. They find that the continuous background absorption can be represented rather well in the region between $\bar{\nu} = 1200$ to $\bar{\nu} = 800\ cm^{-1}$ by a transmission law of the form

$$T(w) = e^{-\left[\frac{4.1 \times 10^4}{(\bar{\nu} - 200)^2} - \frac{3.7 \times 10^3}{(\bar{\nu} - 1550)^2}\right] w} \qquad (3.7.1)$$

The Nepier path R_N according to (3.7.1) lies between 10 and 15 centimeters of precipitable water. This shows that between the strong water vapour and carbon dioxide absorption bands in the infrared, there are windows where signals could penetrate tens of centimeters of precipitable water with little reduction due to absorption and often even less reduction due to aerosol scattering. This is enough to encourage anyone who wants to use lasers for long range communications because it shows that while attenuation is on the whole quite severe, there are circumstances in which range is essentially not limited by the atmospheric scattering or absorption.

3.8 Comments and Conclusions

The comment in previous paragraphs can be condensed into a rough guiding rule for laser communications as follows. Selective laser signals can be transmitted to any terrestrial station that can be seen. This statement can be made rather far reaching by careful interpretation of key words.

such as, selective; signals; transmitted; terrestrial; stations; and seen. The word seen, for example, need not imply naked eye vision. High light-gathering power and high resolution optics could be envisioned. The eye itself could be replaced by sensors which are more sensitive or more discerning in some respects. The light need not be perceptible to the human eye. The statement conveys a good feeling for the capabilities and limitations of the laser despite the large margin for special interpretation left in it. It would have been easy to formulate the statement without the present report. The present report, however, is helpful in making the necessary interpretations and many have been discussed here, some in detail, some by way of a brief phrase or remark. The exposition of the subject is inherently difficult and the literature is hard to gather and hard to read. Many widely-held notions are dubious or misleading or simply incorrect. Now that lasers add greatly to the practical importance of elucidating the details of atmospheric attenuation, it may be expected that clear, concise and sound expositions of the topic will become available and a convenient, useful terminology will arise together with an adequate summary of numerical data in this field.

3.9 Abstract

Small angle spreading, aerosol scattering and molecular absorption are considered the important mechanisms for the weakening of a laser beam in the open atmosphere. Three different transmission laws are worked out for these three mechanisms. Both the physical principles and the numerical values encountered in the lower atmosphere are discussed and illustrated. Random density fluctuations in the turbulent atmosphere are discussed as the cause of small angular deflections in a narrow pencil of light. Beam attenuation due to atmospheric aerosol scattering is treated for an aerosol size

distribution described by the sum of two inverse powers of the droplet radius. Laser beams can help find the parameters of such distributions. Molecular absorption is examined in terms of the narrow infrared lines of water vapour. An effort is made to present this difficult topic in as simple and useful a form as is compatible with the observational material. The formulae are designed to make it possible to estimate in detail how the atmosphere would weaken a laser beam under a wide variety of conditions. It is found that some effects are serious even at short ranges of a few meters, while in favourable circumstances, laser signals would not be drastically attenuated out to any practical distance in the lower atmosphere.